Using the Definite Integral to Measure Work

Work, measured in either joules(Newton-meters) or foot-pounds, is the product of the force necessary to move and object and the displacement or distance of the object caused by the force. It is given by the equation $W = F \cdot d$

If a 10 lb block is pushed 4 feet across a floor, the work done in pushing the block would be 40 foot-lbs.

If a 20 kg block is pushed 3 meters across a floor, the work done in pushing the block would be $W = F \cdot d$ = (20 kg·9.8 m/s^2)(3 meters) = 588 Newton-meters.

Suppose that the force required to move an object x meters across the floor decreases as the object gains speed so that the force can be given by the function $F(x) = Ce^{-0.43x}$.
If the force required to budge the object from rest is 49 N, find C.

C = 49

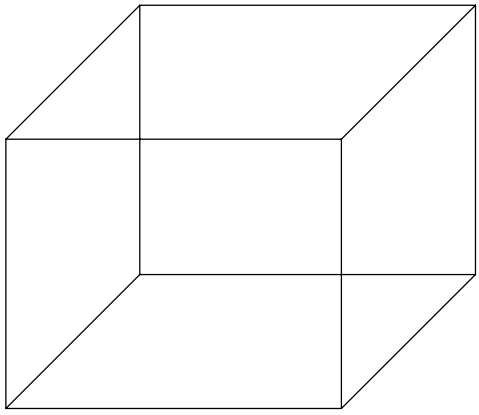
How much work is done moving the object 6 meters?

At any point x, the force required is $49e^{-0.43x}$ and the over a small distance Δx the work done would be $W = 49e^{-0.43x} \cdot \Delta x$. Over a distance from 0 to 6 meters we can approximate it by adding all the small Δx partitions so that we get $\sum_{n=1}^{k} 49e^{-0.43x} \cdot \Delta x$. As k goes to infinity, this summation becomes $\int_0^6 49e^{-0.43x}\,dx \approx 105.32$ joules (or Newton-meters)

A rectangular tank 15 meters long, 10 meters wide, and 9 meters deep is filled with water. The density of water is 1000 kg/m^3. If the water is pumped out through the top of the tank how much work is done emptying the tank?

$F = ma = mg$

$m = Density \cdot Volume$



The force required to move one ‘slab’ of water is
$F = DVg = (1000\ \text{kg/m}^3)(15 \cdot 10 \cdot \Delta y)(9.8\ \text{m/s}^2)$

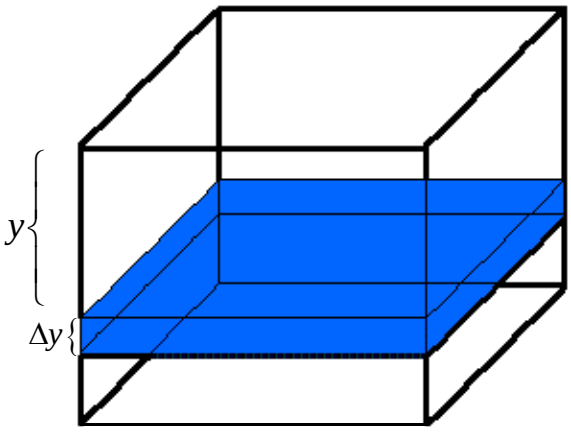
The work done in moving that slab of water to the top is $F \cdot \textit{distance}$

$W = (1000\ \text{kg/m}^3)(15 \cdot 10 \cdot \Delta y)(9.8\ \text{m/s}^2)(y\ \text{meters})$

$$W = 1,470,000 \int_0^9 y\ dy$$

$$= 1{,}470{,}000 \left[\frac{y^2}{2} \right]_0^9$$

$= 59{,}535{,}000$ joules



Name_____

Work

$W = Fd$ ➔ Work = Force x distance ➔ W = $\sum F(x) \cdot \Delta x$ ➔ $\int_a^b F(x)\,dx$

Units for Work are either foot-lbs or Newton-meters(aka joules)

Hooke's Law: $f(x) = kx$ or the amount of force required to stretch a spring x meters is proportional to x with k as the spring constant.

Don't forget: $F = ma$ $m = D \cdot V$ (mass = Density x Volume)

1) Find the amount of work done in lifting a 1.2 kg book off the floor and onto a table that is 0.7 meters high.

2) How much work is done lifting a 20 lb weight 6 feet off the ground?

3) When a particle is located a distance of x feet from the origin, a force of $3x^2$ pounds acts on it. Find the work done moving the particle 5 feet from the origin.

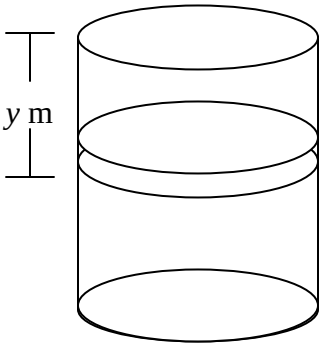
4) A 200 lb. cable measuring 100 feet in length is raised to the top of a building. How much work is done lifting the cable?

5) A heavy rope, 25 m long weighs 200 g/cm and hangs off the edge of a building. How much work is done pulling the rope to the top of the building?

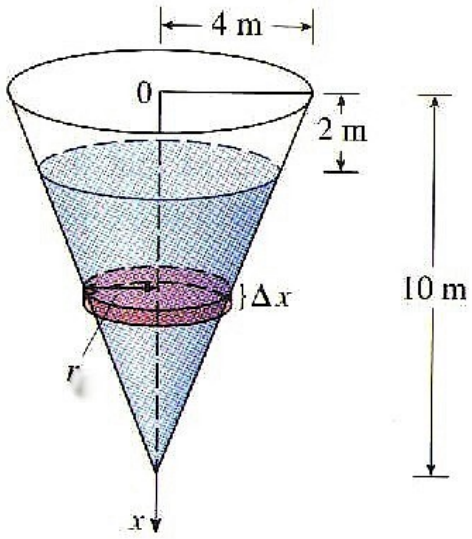
6) A force of 20 N is required to hold a spring that has been stretched from its natural length of 10 cm to a length of 15 cm. How much work is done in stretching the spring from 15 cm to 18 cm?

7) A force of 40 lbs is required to hold a spring that has been stretched from its natural length of 6 inches to a length of 9 inches. How much work is done in stretching the spring from 9 to 12 inches?

8) A cylindrical tank of height 15 meters and a radius of 5 m is filled with water. The water will be pumped through a pipe attached to the top of the tank. Using the fact that the density of water is 1000 kg/m^3, find the amount of work required to empty the tank.



9) An inverted conical tank has a height of 10 m and a radius of 4 m. It is filled with water to a height of 8 m. Find the work required to empty the tank by pumping all of the water to the top of the tank. (The density of water is 1000 kg/m^3)



10) A bucket weighing 5 kg is filled with 10 kg of water. It is attached to a rope that weighs 1 kg and is being lifted to a height of height of 20 meters. While the bucket is rising it is slowly leaking water so that it is empty at the moment it reaches the top. Find the work done

a) lifting the bucket alone.

b) lifting the rope alone

c) lifting the water alone.