Q-5 Homework: p. 590, #1-37

1. $\cos^2 2\theta - \sin^2 2\theta = \frac{\sqrt{3}}{2}$ in $\theta \in [0; 360^\circ]$

$\cos 2[2\theta] = \frac{\sqrt{3}}{2}$

$\cos 4\theta = \frac{\sqrt{3}}{2}$

$4\theta = \pm 30^\circ \pm 360^\circ n$

$\theta = \pm 7.5^\circ \pm 90^\circ n$ $\quad 7.5^\circ$ $\quad 352.5^\circ$

$\theta \in \{7.5^\circ, 82.5^\circ, 97.5^\circ, 172.5^\circ, 187.5^\circ, 262.5^\circ, 277.5^\circ, 352.5^\circ\}$

2. $3 - 3\sin x - 2\cos^2 x = 0$ in $x \in [-2\pi, 2\pi]$

$3 - 3\sin x - 2(1 - \sin^2 x) = 0$

$3 - 3\sin x - 2 + 2\sin^2 x = 0$

$2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin x - 1)(\sin x - 1) = 0$

$2\sin x - 1 = 0$ or $\sin x - 1 = 0$

$2\sin x = 1$ $\quad \sin x = 1$

$\sin x = \frac{1}{2}$

$x = \begin{Bmatrix} \frac{\pi}{6} \pm 2\pi n \\ \frac{5\pi}{6} \pm 2\pi n \end{Bmatrix}$ or $x = \left\{\frac{\pi}{2} \pm 2\pi n\right\}$

$x \in \left\{-\frac{11\pi}{6}, -\frac{3\pi}{2}, -\frac{7\pi}{6}, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}\right\}$

3. $\sin 3\theta \cos 12^\circ - \cos 3\theta \sin 12^\circ = \frac{1}{2}$ in $\theta \in [0^\circ, 360^\circ)$

$\sin(3\theta - 12^\circ) = \frac{1}{2}$

$3\theta - 12^\circ = \begin{Bmatrix} 30^\circ \pm 360^\circ n \\ 150^\circ \pm 360^\circ n \end{Bmatrix}$

$3\theta = 42^\circ \pm 360^\circ n$
$162^\circ \pm 360^\circ n$

$\theta \in \{14^\circ, 54^\circ, 134^\circ, 174^\circ, 254^\circ, 294^\circ$

4 $\sec\left(x-\frac{\pi}{4}\right) = 2 + 2\sec\left(x-\frac{\pi}{4}\right)$ in $x \in (0, 2\pi]$

$-\sec\left(x-\frac{\pi}{4}\right) = 2$

$\sec\left(x-\frac{\pi}{4}\right) = -2$

$\cos\left(x-\frac{\pi}{4}\right) = \frac{-1}{2}$

$x \in \left\{\frac{11\pi}{12}, \frac{19\pi}{12}\right\}$

$x-\frac{\pi}{4} = \pm\frac{2\pi}{3} \pm 2\pi n$

$x = \left\{\begin{matrix}\frac{11\pi}{12} \pm 2\pi n \\ \frac{19\pi}{12} \pm 2\pi n\end{matrix}\right\}$

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5. $2\cos^2\left(\frac{1}{2}x\right) - 2 = 3\cos x$ in $x \in (-\pi, \pi]$

$2\left[\frac{1}{2}(1+\cos x)\right] - 2 = 3\cos x$

$\cos x - 1 = 3\cos x$

$-1 = 2\cos x$

$\frac{-1}{2} = \cos x$

$x = \left\{\pm\frac{2\pi}{3} \pm 2\pi n\right\}$

$x \in \left\{-\frac{2\pi}{3}, \frac{2\pi}{3}\right\}$

6. $\cos 4\theta - \cos 2\theta = 0$ in $\theta \in [0, 360^\circ]$

$\cos 2[2\theta] - \cos 2\theta = 0$

$\cos^2 2\theta - \sin^2 2\theta - \cos 2\theta = 0$

$\cos^2 2\theta - (1 - \cos^2 2\theta) - \cos 2\theta = 0$

$2\cos^2 2\theta - \cos 2\theta - 1 = 0$

$(2\cos 2\theta + 1)(\cos 2\theta - 1) = 0$

$2\cos 2\theta + 1 = 0$ or $\cos 2\theta - 1 = 0$

$\cos 2\theta = \frac{-1}{2}$ or $\cos 2\theta = 1$

$2\theta = \pm 120 \pm 360n$ $\quad 2\theta = 0 \pm 360n$

$\theta = \pm 60 \pm 180n$ $\quad \theta = 0^\circ \pm 180n$

~~-60~~, 120, 300, ~~420~~ $\quad$ 0 or ~~180~~ 360

~~60~~, 240, ~~420~~

$\theta \in \{0^\circ, 60^\circ, 120^\circ, 180^\circ, 240^\circ, 300^\circ, 360^\circ\}$

7. $4\sin x \cos x = \sqrt{3}$ in $x \in [0, 2\pi]$

$2(2\sin x \cos x) = \sqrt{3}$

$2\sin x \cos x = \frac{\sqrt{3}}{2}$

$\sin(2x) = \frac{\sqrt{3}}{2}$

$2x = \left\{ \frac{\pi}{3} \pm 2\pi n, \ \frac{2\pi}{3} \pm 2\pi n \right\}$

$x = \left\{ \frac{\pi}{6} \pm \pi n, \ \frac{\pi}{3} \pm \pi n \right\}$

$\pm \frac{\pi}{2}$

$\frac{1}{6}, \frac{7}{6}$

$x \in \left\{ \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3} \right\}$

8. $\sin^2 x - \cos^2 x = \frac{\sqrt{3}}{2}$ in $x \in [0, 2\pi]$

$\cos^2 x - \sin^2 x = -\frac{\sqrt{3}}{2}$

$\cos 2x = -\frac{\sqrt{3}}{2}$

$2x = \pm \frac{5\pi}{6} \pm 2\pi n$

$x = \pm \frac{5\pi}{12} \pm \pi n$

$\frac{5}{12} \quad \frac{17}{12}$

$\frac{7\pi}{12}, \frac{19\pi}{12}$

$x \in \left\{ \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12} \right\}$

9. $\cos\theta\cos 20° - \sin\theta\sin 20° = \frac{1}{\sqrt{2}}$ in $\theta \in [0°, 360°]$

$\cos(\theta + 20°) = \frac{1}{\sqrt{2}}$

$\theta + 20° = \pm 45° \pm 360°n$

$\theta = 25° \pm 360°n$

$295° \pm 360°n$

$\theta = \{25°, 295°\}$

10. $2\cos^2 x = \sqrt{3} + 2\sin^2 x$ in $x \in [-180°, 0]$

$2\cos^2 x - 2\sin^2 x = \sqrt{3}$

$2(\cos^2 x - \sin^2 x) = \sqrt{3}$

$\cos^2 x - \sin^2 x = \frac{\sqrt{3}}{2}$

$\cos 2x = \frac{\sqrt{3}}{2}$

$2x = \pm 30° \pm 360°n$

$x = \pm 15° \pm 180°n$

$-15, 165$

$x \in \{-165°, -15°\}$

11. $\tan^4 x - 4\tan^2 x + 3 = 0$ in $x \in \left[\frac{-\pi}{4}, \frac{\pi}{4}\right)$

$(\tan^2 x - 3)(\tan^2 x - 1) = 0$

$\tan^2 x = 3$ or $\tan^2 x = 1$

$\tan x = \pm\sqrt{3}$ $\quad$ $\tan x = \pm 1$

$x \in \left\{\frac{-\pi}{4}\right\}$

$x = \left\{\begin{matrix}\frac{\pi}{3} \pm \pi n \\ \frac{5\pi}{3} \pm \pi n \\ -\frac{1}{3}\end{matrix}\right\}$ $\quad$ $x = \left\{\begin{matrix}\frac{\pi}{4} \pm \pi n \\ \frac{3\pi}{4} \pm \pi n \\ -\frac{1}{4}\end{matrix}\right\}$

12. $\sin^2 3t - 2\sin\frac{3}{2}t\cos\frac{3}{2}t - 2 = 0$ in $t \in [0°, 360°)$

$\sin^2 3t - \sin 3t - 2 = 0$

$(\sin 3t - 2)(\sin 3t + 1) = 0$

$\sin 3t = 2$ $\quad$ $\sin 3t = -1$

no solution $\quad$ $3t = 270° \pm 360°n$

$90° \pm 120°n$

$90°, 210°, 330°$

$t \in \{90°, 210°, 330°\}$

13. $3 - 3\sin x - 2\cos^2 x = 0$ in $x \in \{\text{All Reals}\}$

$3 - 3\sin x - 2(1 - \sin^2 x) = 0$

$3 - 3\sin x - 2 + 2\sin^2 x = 0$

$2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin x - 1)(\sin x - 1) = 0$

$2\sin x = 1$ $\quad$ $\sin x = 1$

$\sin x = \frac{1}{2}$

$x = \left\{\begin{matrix}\frac{\pi}{6} \pm 2\pi n \\ \frac{5\pi}{6} \pm 2\pi n\end{matrix}\right\}$ $\quad$ $x = \left\{\frac{\pi}{2} \pm 2\pi n\right\}$

$x = \left\{\begin{matrix}\frac{\pi}{6} \pm 2\pi n \\ \frac{\pi}{2} \pm 2\pi n \\ \frac{5\pi}{6} \pm 2\pi n\end{matrix}\right\}$

14. $2\cot^2 x - 5\csc x + 5 = 0$ on $x \in [-2\pi, 2\pi)$

$2(\csc^2 x - 1) - 5\csc x + 5 = 0$

$2\csc^2 x - 2 - 5\csc x + 5 = 0$

$2\csc^2 x - 5\csc x + 3 = 0$

$(2\csc x - 3)(\csc x - 1) = 0$

$2\csc x = 3 \qquad \csc x = 1$

$\csc x = \frac{3}{2} \qquad \sin x = 1$

$\sin x = \frac{2}{3}$

$x \in \left\{ -\frac{3\pi}{2}, \frac{\pi}{2}, -5.553, -3.871, .730, 2.412 \right\}$

$x = \begin{cases} .730 \pm 2\pi n \\ 2.412 \pm 2\pi n \end{cases} \qquad x = \left\{ \frac{\pi}{2} \pm 2\pi n \right\}$

$-\frac{3\pi}{2}, \frac{\pi}{2}$

$-5.553, -3.871$

15. $\tan^4 x - \sec^4 x = -3$ on $x \in [-45°, 0°]$

$(\tan^2 x - \sec^2 x)(\tan^2 x + \sec^2 x) = -3$

$(\sec^2 x - \tan^2 x)(\tan^2 x + \sec^2 x) = 3$

$\tan^2 x + \sec^2 x = 3$

$\tan^2 x + \tan^2 x + 1 = 3$

$2\tan^2 x = 2$

$\tan^2 x = 1$

$\tan x = \pm 1$

$x \in \{-45°\}$

$\tan x = 1 \qquad \tan x = -1$

$x = 45° \pm 180°n \qquad x = -45° \pm 180°n$

16. $\tan^2 x - \sec^2 x - \sec x - 2 = 0$ in $x \in [-2\pi, 2\pi]$

$\sec^2 x - 1 - \sec^2 x - \sec x - 2 = 0$

$-\sec x - 3 = 0$

$\sec x = -3$

$\cos x = \frac{-1}{3}$ $\quad x \in \{-1.911, 1.911\}$

$x = \pm 1.911 \pm 2\pi n$

17. $\sin^2 A - 2\sin(3A+\pi) = -\cos^2 A$ in $A \in (\pi, 3\pi)$

$\sin^2 A + \cos^2 A - 2\sin(3A+\pi) = 0$

$1 - 2\sin(3A+\pi) = 0$

$-2\sin(3A+\pi) = -1$

$\sin(3A+\pi) = \frac{1}{2}$

$3A + \pi = \begin{cases} \frac{\pi}{6} \pm 2\pi n \\ \frac{5\pi}{6} \pm 2\pi n \end{cases}$

$3A = \begin{cases} \frac{-5\pi}{6} \pm 2\pi n \\ \frac{11\pi}{6} \pm 2\pi n \end{cases}$

$A = \begin{cases} \frac{-5\pi}{18} \pm \frac{2\pi}{3} n \\ \frac{11\pi}{18} \pm \frac{2\pi}{3} n \end{cases}$

$A \in \left\{ \frac{19\pi}{18}, \frac{23\pi}{18}, \frac{31\pi}{18}, \frac{35\pi}{18}, \frac{43\pi}{18}, \frac{47\pi}{18} \right\}$

~~$\frac{7\pi}{18}$~~, $\frac{19\pi}{18}$, $\frac{31\pi}{18}$, $\frac{43\pi}{18}$, ~~$\frac{55\pi}{18}$~~

~~$\frac{11\pi}{18}$~~, $\frac{23\pi}{18}$, $\frac{35\pi}{18}$, $\frac{47\pi}{18}$, ~~$\frac{59\pi}{18}$~~

16. $2\sin(2x-30^\circ)\cos(2x-30^\circ) = -\frac{\sqrt{3}}{2}$ in $x \in [0^\circ, 720^\circ]$

$$\sin 2(2x-30^\circ) = -\frac{\sqrt{3}}{2}$$

$$\sin(4x-60^\circ) = -\frac{\sqrt{3}}{2}$$

$$4x-60^\circ = 240^\circ \pm 360^\circ n$$
$$300^\circ \pm 360^\circ n$$

$$4x = 300^\circ \pm 360^\circ n$$
$$360^\circ \pm 360^\circ n$$

$$x = 75^\circ \pm 90^\circ n$$
$$90^\circ \pm 90^\circ n$$

$75, 165, 255^\circ, 345, 435, 525, 615, 705^\circ$

$90, 180, 270^\circ, 360, 450, 540, 630, 720^\circ$

$$x = \begin{cases} 75^\circ, 90^\circ, 165^\circ, 180^\circ, \\ 255^\circ, 270^\circ, 345^\circ, 360^\circ \\ 435^\circ, 450^\circ, 525^\circ, 540^\circ, \\ 615^\circ, 630^\circ, 705^\circ, 720^\circ \end{cases}$$

19 $\sec^2(x-4) + \sin(3x) = 1 + \tan^2(x-4)$ in $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right)$

~~$\sec^2(x-4) + \sin(3x) = \sec^2(x-4)$~~

$$\sin(3x) = 0$$

$$3x = 0 \pm 2\pi n$$
$$\pi \pm 2\pi n$$

$$x = \begin{cases} 0 \pm \frac{2\pi}{3} n \\ \frac{\pi}{3} \pm \frac{2\pi}{3} n \end{cases}$$

$$x \in \left\{-\frac{\pi}{3}, \frac{\pi}{3}\right\}$$

20. $\cos 2x \sin x + \sin 2x \cos x - 2\sin 3x = \frac{1}{2}$ in $x \in [0, 2\pi)$

$\sin(2x + x) - 2\sin 3x = \frac{1}{2}$

$\sin 3x - 2\sin 3x = \frac{1}{2}$

$-\sin 3x = \frac{1}{2}$

$\sin 3x = -\frac{1}{2}$

$3x = \begin{cases} \frac{11\pi}{6} \pm 2\pi n \\ \frac{7\pi}{6} \pm 2\pi n \end{cases}$

$x = \begin{cases} \frac{11\pi}{18} \pm \frac{2\pi n}{3} \\ \frac{7\pi}{18} \pm \frac{2\pi n}{3} \end{cases}$

$x \in \left\{ \frac{7\pi}{18}, \frac{11\pi}{18}, \frac{19\pi}{18}, \frac{23\pi}{18}, \frac{31\pi}{18}, \frac{35\pi}{18} \right\}$

$\frac{11\pi}{16}, \frac{23\pi}{18}, \frac{35\pi}{16}$

$\frac{7\pi}{16}, \frac{19\pi}{16}, \frac{31\pi}{18}$

21. $\csc 2x + \frac{1}{\sin x \cos x} = 6$ in $x \in [0, \pi]$

$\frac{1}{\sin 2x} + \frac{1}{\sin x \cos x} = 6$

$\frac{1}{2\sin x \cos x} + \frac{2}{2\sin x \cos x} = 6$

$\frac{3}{2\sin x \cos x} = 6$

$\frac{1}{2\sin x \cos x} = 2$

$\sin 2x = \frac{1}{2}$

$\alpha x = \frac{\pi}{6} \pm 2\pi n$

$\frac{5\pi}{6} \pm 2\pi n$

$x = \frac{\pi}{12} \pm \pi n$

$\frac{5\pi}{12} \pm \pi n$

$x \in \left\{ \frac{\pi}{12}, \frac{5\pi}{12} \right\}$

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22. $\frac{1-\cos x}{\sin x} = \sqrt{3}$ in $x \in (0, 2\pi)$

$\tan\left(\frac{1}{2}x\right) = \sqrt{3}$

$\frac{1}{2}x = \left\{\frac{\pi}{3} \pm \pi n\right\}$

$x = \left\{\frac{2\pi}{3} \pm 2\pi n\right\}$

$x \in \left\{\frac{2\pi}{3}\right\}$

NO ACC

~~35~~ $\frac{1-\tan^2 x}{2\tan x} + \frac{\cos^2 x - \sin^2 x}{2\sin x \cos x} = 1$ in $x \in (0, 2\pi)$

$\cot 2x + \frac{\cos 2x}{\sin 2x} = 1$

$2\cot 2x = 1$

$\cot 2x = \frac{1}{2}$

$\tan 2x = 2$

$2x = \{1.107 \pm \pi n\}$

$x = \{.554 \pm \frac{\pi}{2} n\}$

$x \in \{.554, 2.125, 3.696, 5.266\}$

2.125, 3.696, 5.266, ~~6.827~~