Solutions

Central Limit Theorem Activity

Example: A measurement from a population has population mean 6 and standard deviation 2. What are the mean and standard error of $\overline{x}$ when $n = 4$? When $n = 100$? When $n = 400$?

$\overline{x} = 6$ for all values of n

$n = 4$ $\quad \sigma_{\overline{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{2}{\sqrt{4}} = 1$

$n = 100$ $\quad \sigma_{\overline{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{2}{\sqrt{100}} = 0.2$

$n = 400$ $\quad \sigma_{\overline{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{2}{\sqrt{400}} = 0.1$

Note that as n approaches infinity (as our sample size approaches the population size) then should approach 0 which means that our sample mean will eventually approach and not vary from our population mean.

1. For the population of farm workers in New Zealand, suppose that weekly income has a distribution that is skewed right with a mean of μ = \$500 (N.Z. dollars) and a standard deviation of σ = \$160. A survey of 100 farm workers is taken, including information on their weekly income.

(a) What are the mean and standard deviation of the sampling distribution of $\overline{x}$?

$\mu_{x} = \underline{500} \qquad \sigma_{\overline{x}} = 160/\sqrt{100} = \underline{16}$

(b) What is the probability that the mean weekly income of these 100 workers is less than \$448?

Using z-scores $\quad P(X < 448) = P\left(Z < \dfrac{448-500}{16}\right) = P(Z < -3.25) = \text{normalcdf}(-1E99, -3.25)$

$= 0.0006$

Using the normal distribution $\quad P(\overline{x} < 448) = \text{normalcdf}(-1E99, 448, 500, 16) = 0.0006$

(c) What is the probability that the mean weekly income of these 100 workers is between \$480 and \$520?

X = weekly income

$P(480 < X < 520) = \text{normal}(480, 520, 500, 16) = 0.7888$

2. The heights of 18-year-old men are approximately normally distributed with mean of 68 inches and standard deviation of 3 inches.
(a) What is the probability that an 18-year-old man selected at random is between 67 and 69 inches tall?

$$P(67 < X < 69) = \text{normalcdf}(67, 69, 68, 3) = 0.261$$

(b) For a sample of 36 18-year-old men, what is the probability that the average of their heights is between 67 and 69 inches?

$$\sigma_{\bar{x}} = 3/\sqrt{36} = 3/6 = \frac{1}{2}$$

$$\text{normalcdf}\left(67, 69, 68, \frac{1}{2}\right) = 0.954$$

3. For people under 50, the level of glucose in the blood (in milligrams per deciliter of blood) after a 12-hour fast have a standard deviation of 25 and a mean of μ. What is the probability that, for a sample of size 49 readings, the sample mean is within 7 of μ? (Hint: Recall the formula for a *z*-score)

$$\frac{\sigma}{\sqrt{n}} = \frac{25}{7}$$

$$\overbrace{-7 < \bar{X} - \mu < 7}$$

Method 1

$$P(-7 < \bar{x} - \mu < 7)$$

noting that $\frac{\bar{X}-\mu}{\sigma_{\bar{x}}} = Z_{\bar{x}}$ we can divide through by $\sigma_{\bar{x}}$ to make it a standard normal probability $P\left(\frac{-7}{\sigma_{\bar{x}}} < \underline{\frac{\bar{X}-\mu}{\sigma_{\bar{x}}}} < \frac{7}{\sigma_{\bar{x}}}\right)$

$$= P\left(\frac{-7}{25/7} < Z_{\bar{x}} < \frac{7}{25/7}\right) = \text{normalcdf}\left(-\frac{49}{25}, \frac{49}{25}\right) = 0.950$$

SN $x = z$

Method 2

$$P(-7 < \underbrace{\bar{X} - \mu} < 7)$$

The normal curve is shaped by σ and μ just tells us the location of the middle value so we can assign $(\bar{X} - \mu)$ any value we want on the calculator

the mean we enter can be any value we want provided the left and right bounds are within 7 of the mean

$$\text{normalcdf}\left(\underset{\mu-7}{-7}, \underset{\mu+7}{7}, \underset{\mu}{0}, 25/7\right) = 0.950$$

Formulae: $\mu_{\bar{X}-\bar{Y}} = \mu_{\bar{X}} - \mu_{\bar{Y}}$ $\qquad$ $\sigma_{\bar{X}-\bar{Y}} = \sqrt{\left(\sigma_{\bar{X}}\right)^2 + \left(\sigma_{\bar{Y}}\right)^2} = \sqrt{\frac{\sigma_1{}^2}{n_2} + \frac{\sigma_2{}^2}{n_2}}$

4) Ava has collected the times it takes students in first period AP Statistics to complete homework assignments. She takes many samples of 9 assignments and creates a sampling distribution for the class and finds the mean to be 22.3 minutes and the standard deviation to be 1.6 minutes. Abby does the same thing for fourth period AP Statistics and finds that with the sample size of 9, her distribution has a mean of 25.4 minutes and standard deviation of 2.0 minutes. Given that both distributions are approximately normal, use this information to find each of the following:

a. Find the mean and standard deviation of the population for 1st period AP Statistics.

$\mu_{1st} = \mu_{\bar{x}_{1st}} = 22.3$ $\qquad$ $\sigma_{\bar{x}_{1st}} = 1.6 = \frac{\sigma_1}{\sqrt{9}} = \frac{\sigma_1}{3} \Rightarrow \sigma_{1st} = 3 \cdot 1.6 = 4.8$

b. Find the mean and standard deviation of the population for 4th period AP Statistics.

$\mu_4 = \mu_{\bar{x}_{4th}} = 25.4$ $\qquad$ $\sigma_{\bar{x}_{4th}} = 2.0 = \frac{\sigma_4}{\sqrt{9}} = \frac{\sigma_4}{3} \Rightarrow \sigma_{4th} = 3(2.0) = 6.0$

c. Find the mean and standard deviation of the difference between the sampling distributions.

$\mu_{4th-1st} = \mu_{4th} - \mu_{1st} = 25.4 - 22.3$

$= 3.1$

$\sigma_{\bar{x}_{4th}-\bar{x}_{1st}} = \sqrt{\frac{\sigma_{4th}^2}{\sqrt{9}} - \frac{\sigma_{1st}^2}{\sqrt{9}}} = \sqrt{(1.6)^2 + (2.0)^2}$

$= 2.561$

d. Find the mean and standard deviation of the difference between the populations.

$\mu_{4th-1st} = \mu_{4th} - \mu_{1st} = 25.4 - 22.3$

$= 3.1$

$\sigma_{4th-1st} = \sqrt{\sigma_4^2 - \sigma_1^2} = \sqrt{9(1.6)^2 + 9(2.0)^2}$

$= 3(2.561)$

$= 7.684$

e. What is the probability that a student from 1st period takes more than 30 minutes to complete an assignment?

$\mu = 22.3$

$\sigma = 4.8$

left $\quad$ right $\quad$ μ $\quad$ σ

normalcdf(30, 1E99, 22.3, 4.8) = 0.543