

Central Limit Theorem Activity

AP Statistics

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Formulae: $\mu_{\bar{x}} = \mu$ $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Example: A measurement from a population has population mean 6 and standard deviation 2. What are the mean and standard deviation of \bar{x} when $n = 4$? When $n = 100$? When $n = 400$?

$$\mu_{\bar{x}} = 6 \text{ for all values of } n$$

$$n = 4 \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{4}} = 1 \qquad n = 100 \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{100}} = 0.2$$

$$n = 400 \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{400}} = 0.1$$

Note that as n approaches infinity (as our sample size approaches the population size) then should approach 0 which means that our sample mean will eventually approach and not vary from our population mean.

- 1) For the population of farm workers in New Zealand, suppose that weekly income has distribution that is skewed right with a mean of $\mu = \$500$ (N.Z. dollars) and a standard deviation of $\sigma = \$160$. A survey of 100 farm workers is taken, including information on their weekly income.
 - a. What are the mean and standard deviation of the sampling distribution of \bar{X} ? Can you assume that this sampling distribution would be approximately normal? Explain.
 - b. What is the probability that the mean weekly income of these 100 workers is less than \$448?
 - c. What is the probability that the mean weekly income of these 100 workers is between \$480 and \$520?

Formulae: $\mu_{\bar{X}-\bar{Y}} = \mu_{\bar{X}} - \mu_{\bar{Y}}$ $\sigma_{\bar{X}-\bar{Y}} = \sqrt{(\sigma_{\bar{X}})^2 + (\sigma_{\bar{Y}})^2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

- 4) Ava has collected the times it takes students in first period AP Statistics to complete homework assignments. She takes many samples of 9 assignments and creates a sampling distribution for the class and finds the mean to be 22.3 minutes and the standard deviation to be 1.6 minutes. Abby does the same thing for fourth period AP Statistics and finds that with the sample size of 9, her distribution has a mean of 25.4 minutes and standard deviation of 2.0 minutes. Given that both distributions are approximately normal, use this information to find each of the following:
- Find the mean and standard deviation of the population for 1st period AP Statistics.
 - Find the mean and standard deviation of the population for 4th period AP Statistics.
 - Find the mean and standard deviation of the difference between the sampling distributions.
 - Find the mean and standard deviation of the difference between the populations.
 - What is the probability that a student from 1st period takes more than 30 minutes to complete an assignment?