Final Exam Standards

1c	Given a point or trig function value and quadrant of the terminal side of an angle, find the exact value of all the trigonometric functions of the angle.
1f	Use exact values from the special triangles to simplify trigonometric expressions
1g	Use a calculator to find approximate trigonometric values for a given angle and approximate angle values for a given trigonometric value.
1j	Model and solve problems involving vectors
2a	Use a graphing calculator to find the graph of a trigonometric equation.
2b	Find the graph from the equation of a sinusoidal without a graphing calculator.
2c	Given a sinusoidal graph or its traits, find its equation.
2d	Given a sinusoidal equation, find values of y from x and vice versa.
2e	Model and Solve Sinusoidal Equations
3b	Factor polynomials using grouping and sum/difference rules
3e	Solve equations involving composite argument rules.
3h	Solve equations involving double angle rules
3i	Solve equations involving half angle rules
4a	Find equations and intercepts of lines from points, slopes, and parallel or perpendicular lines.
4b	Find equations, zeros, vertex, and range of a parabola.

Pre-Calculus Practice Final Exam

Find the values of x for which:

1) $x^2 - 19x + 60 = 0$

$x^2 - 15x - 4x + 60 = 0$

$x(x-15) - 4(x-15) = 0$

$(x-15)(x-4) = 0$

$x = 4, 15$

2) $3x^2 + 5x - 28 = 0$ $\Rightarrow$ difference of $+5$ product of $3(-28) = -84$

$3x^2 + 12x - 7x - 28 = 0$

$3x(x+4) - 7(x+4) = 0$

$(x+4)(3x-7) = 0$

$x = -4, \frac{7}{3}$

Simplify

3) $\dfrac{x^2+4x+3}{3x^3+3x^2}\cdot\dfrac{12x^3+6x^2+3x}{x^2-9}\div\dfrac{8x^3-1}{x^2-1}$

$$\frac{\cancel{(x+3)}(x+1)}{3x^2\cancel{(x+1)}}\cdot\frac{\cancel{3x}\cancel{(4x^2+2x+1)}}{(x-3)\cancel{(x+3)}}\cdot\frac{(x-1)\cancel{(x+1)}}{(2x-1)(4x^2+2x+1)}$$

$$\frac{(x+1)(x-1)}{x(x-3)(2x-1)}$$

4) $\dfrac{x^3-5x^2+2x-10}{x^2-25}\div\dfrac{x^3+2x}{x^2}$

$$\frac{x^2(x-5)+2(x-5)}{(x-5)(x+5)}\cdot\frac{x^2}{x(x^2+2)}$$

$$\frac{\cancel{(x^2+2)}\cancel{(x-5)}}{\cancel{(x-5)}(x+5)}\cdot\frac{x^{\cancel{2}}}{\cancel{x}\cancel{(x^2+2)}}=\frac{x}{(x+5)}$$

5) Find the equation of the line with an $\underbrace{x\text{-intercept of 2}}$ and perpendicular to one through the points (–4, –1) and (–2, –7)

$(2,0)$

Points

(–4, –1) and (–2, –7)

$$m=\frac{-7-(-1)}{-2-(-4)}=\frac{-6}{2}$$

$$=-3$$

$$m_\perp=\frac{1}{3}\Rightarrow y-0=\frac{1}{3}(x-2)$$

6) Find the vertex and x and y intercepts of the given parabola and graph on the given axes

$$y = 3x^2 - 12x - 15$$

y-int $(0,-15)$

x-int

$0 = 3x^2 - 12x - 15$

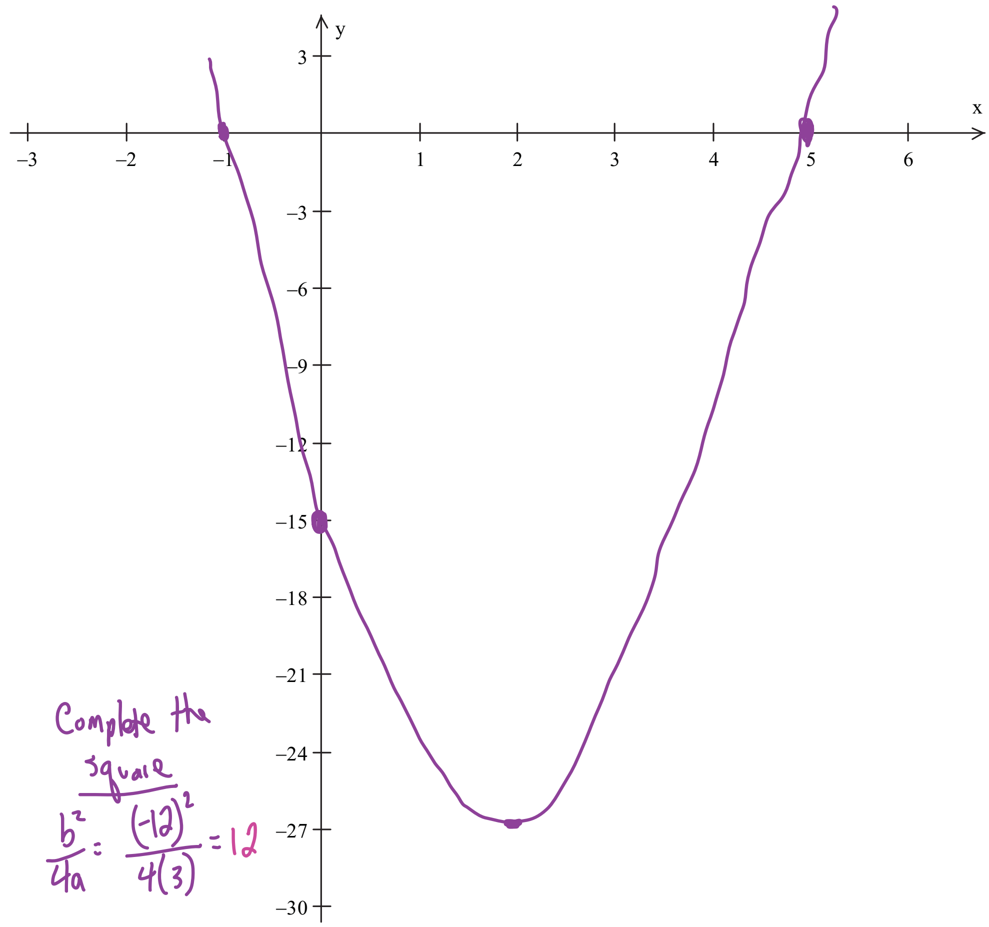
$= 3x^2 - 15x + 3x - 15$

$= 3x(x-5) + 3(x-5)$

$= (x-5)(3x+3)$

$= 3(x-5)(x+1)$

$(-1,0)\ (5,0)$



Vertex

$y = 3x^2 - 12x + 12 - 15 - 12$

$= 3(x^2 - 4x + 4) - 15 - 12$

$= 3(x-2)^2 - 27$

$(h,k): (2,-27)$

Complete the square

$\frac{b^2}{4a} = \frac{(-12)^2}{4(3)} = 12$

Hint: another short cut to finding the vertex is to note that h is always half way between the two x-intercepts

7) Solve for all possible values of x

$2\left(\sin x \cos x = \frac{\sqrt{3}}{4}\right)$

$2 \sin x \cos x = \frac{\sqrt{3}}{2}$

$\sin 2x = \frac{\sqrt{3}}{2}$

$2x = 60° \pm 360°n$

$120° \pm 360°n$

$x = 30° \pm 180°n$

$60° \pm 180°n$

8) $\sqrt{\frac{1-\cos x}{2}} = -\frac{\sqrt{3}}{2}$ $540° \le x \le 630°$

$\sin \frac{1}{2}x = -\frac{\sqrt{3}}{2}$

$\frac{1}{2}x = 300° \pm 360°n$
$240°$

$x = 600° \pm 720°n$
$480°$

9) Solve for all possible values of x $\sin 2x \cos 5° - \cos 2x \sin 5° = \frac{\sqrt{2}}{2}$

$\sin(2x-5) = \frac{\sqrt{2}}{2}$

$2x - 5 = 45° \pm 360°n$
$135° \pm 360°n$

$2x = 50°$, $140°$ $\pm 360°n$

$x = 25°$, $70°$ $\pm 180°n$

10) If (–9, –40) is a point on the terminal side of angle A, find the exact values of

$\cos A = -\frac{9}{41}$ $\sec A = -\frac{41}{9}$ $x = -9 \quad y = -40$

$\sin A = -\frac{40}{41}$ $\csc A = -\frac{41}{40}$ $x^2 + y^2 = r^2$

$\tan A = \frac{40}{9}$ $\cot A = \frac{9}{40}$ $9^2 + 40^2 = r^2$

$r = 41$

11) Use the results from #10 to solve for the following

a) $\sin 2A = 2\sin A\cos A$

$= 2\left(-\frac{40}{41}\right)\left(-\frac{9}{41}\right)$

$= \frac{720}{1681}$

b) $\cos 2A = \cos^2 A - \sin^2 A$

$= \left(-\frac{9}{41}\right)^2 - \left(-\frac{40}{41}\right)^2$

$= \frac{81}{1681} - \frac{1600}{1681} = -\frac{1519}{1681}$

c) The quadrant in which $2A$ terminates with explanation

QII because $\sin A > 0$ and $\cos A < 0$

12) Find *all* values for each of the following (in degrees).

a) $\sin^{-1}(0.656059029) =$ $41° \pm 360°n$, $(180 - 41°) \pm 360°n$ $=$ $41° \pm 360°n$, $139° \pm 360°n$

b) $\tan^{-1}(-1.664279482) = -59° \pm 180°n$

c) $\cos\theta = -0.5877852523$

$\theta = \cos^{-1}(-0.5877852523) = \pm 126° \pm 360°n$

13) If $\sec B = -\dfrac{\sqrt{26}}{5}$ and is in Quadrant III, find the following:

$\sin B = -\dfrac{1}{\sqrt{26}}$ $\csc B = -\sqrt{26}$

$\cos B = -\dfrac{5}{\sqrt{26}}$ $\sec B = -\dfrac{\sqrt{26}}{5}$

$\tan B = \dfrac{1}{5}$ $\cot B = 5$

$x = -5$

$r = \sqrt{26}$

$x^2 + y^2 = r^2$

$(-5)^2 + y^2 = 26$

$25 + y^2 = 26$

$y^2 = 1$

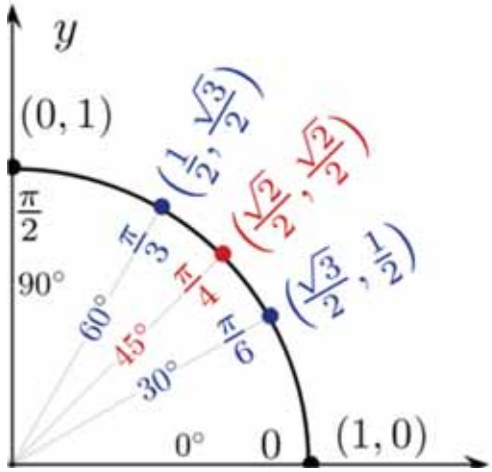
$y = \pm 1 \Rightarrow y = -1$ because QIII

Find angle B rounded to three decimal places (in radians).

$B =$ -2.944 or 3.339 radians

$\cos^{-1}\left(-\frac{5}{\sqrt{26}}\right) = \pm 2.944$

14) Find the *exact* values of each of the following expressions. Show all work and use only the unit circle to find your trig values



a) $\cos\left(\frac{5\pi}{6}\right) - \sin\left(\frac{2\pi}{3}\right)$

$$-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} = -\frac{2\sqrt{3}}{2}$$

$$= -\sqrt{3}$$

b) $\tan^2\left(\frac{5\pi}{4}\right) - \sec^2\left(\frac{5\pi}{4}\right)$

$$(1)^2 - \left(\frac{1}{-\frac{\sqrt{2}}{2}}\right)^2$$

$$1 - \frac{1}{\frac{2}{4}} = 1 - \frac{4}{2}$$

$$1 - 2 = -1$$

c) $\csc^2\left(\frac{\pi}{4}\right) - \cos\left(\frac{5\pi}{3}\right)$

$$\left(\frac{1}{\frac{\sqrt{2}}{2}}\right)^2 - \left(\frac{1}{2}\right) = 2 - \frac{1}{2} = \frac{3}{2}$$

15) Julia, being chased by Shay and Carter who are offended by something she said, runs 27 feet at 57°, turns and runs 32 feet at 221°, and then turns again to run 18 feet at 187°. Find the distance and direction that she has run from where he started.

add each column

$$(27\cos 57°)i + (27\sin 57°)j$$

$$(32\cos 221)i + (32\sin 221°)j$$

$$(18\cos 187°)i + (18\sin 187°)j$$

Recall how to use the store feature on your calculator as well as $\boxed{\text{2nd}}$ $\boxed{\text{Enter}}$

(See Vector Screencasts)

$$\vec{V} = (-27.311\ldots)i + (-0.5434\ldots)j$$

$$|\vec{V}| = \text{magnitude} = \sqrt{(-27.311\ldots)^2 + (-0.5434\ldots)^2}$$

$$= 27.317 \text{ ft}$$

Direction: $\theta = \pm\cos^{-1}\left(\frac{-27.311\ldots}{27.317\ldots}\right) = -178.860°$

y-component is $-0.5434\ldots$ so this is $-$

$\frac{\text{x-component}}{\text{magnitude}} = \frac{x}{r}$

16) Given that $\vec{u} = 5\vec{i} - 36\vec{j}$ and $\vec{v} = 8\vec{i} - 49\vec{j}$, find each of the following:

a. $\vec{u} + \vec{v}$

$$5i - 36j + 8i - 49j = (5+8)i + (-36-49)j = 13i - 85j$$

b. $\underbrace{|\vec{u} + \vec{v}|}_{\text{Pythag.}} = \sqrt{13^2 + (-85)^2} = \sqrt{7394} \approx 85.988$

Not absolute value of components

Find two periods for each function. Label and mark the axes appropriately

v-shift amplitude horizontal shift

17) $y = 3 + 5\cos\left[\frac{\pi}{6}(x-2)\right]$

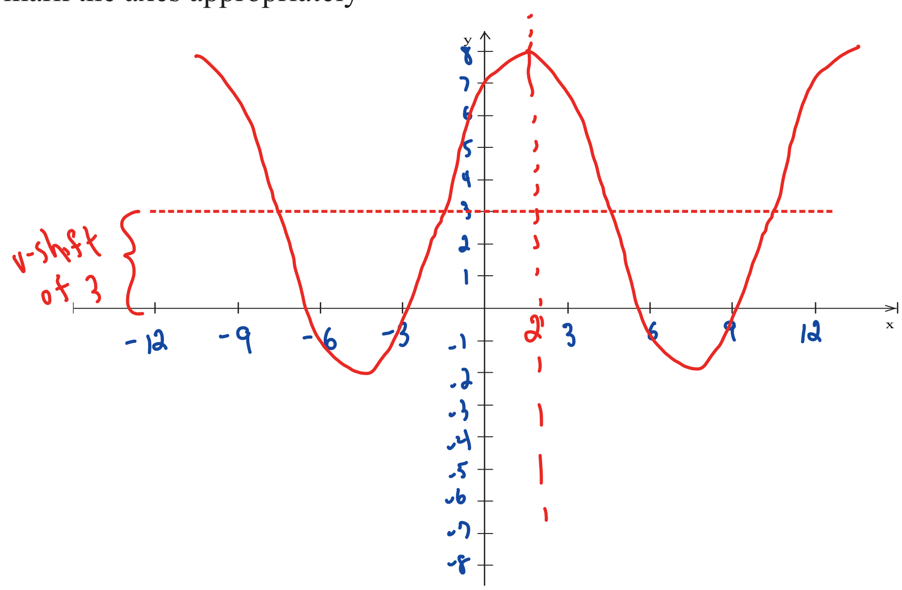
$\frac{2\pi}{\pi/6}$

amplitude:

period $= 12 =$ period

vertical shift

horizontal shift



Max = v-shift + amp = 3+5 = 8

min = v-shift − amp = 3−5 = −2

flip the graph upside down

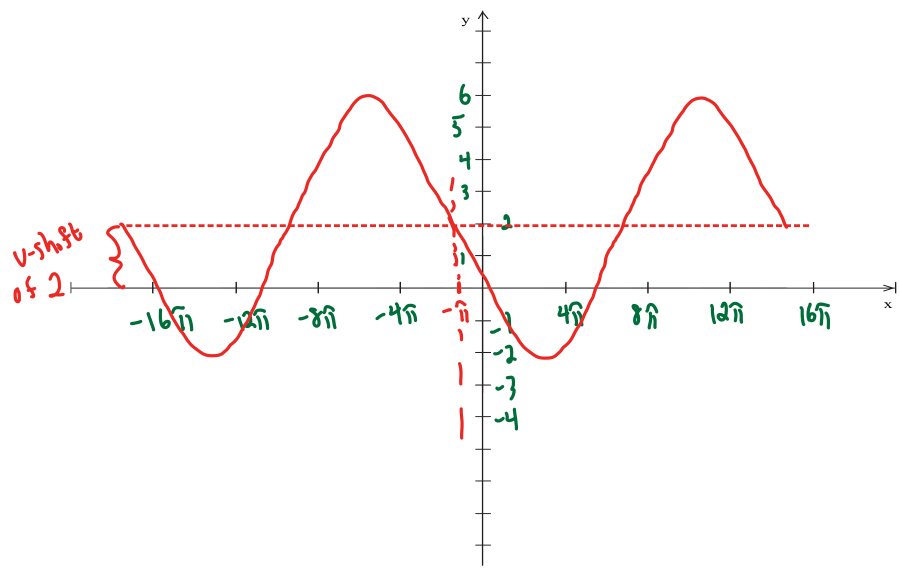
18) $f(x) = 2 - 4\sin\left[\frac{1}{8}(x+\pi)\right]$

amplitude: 4

period $\frac{2\pi}{1/8} = 16\pi$

vertical shift = 2

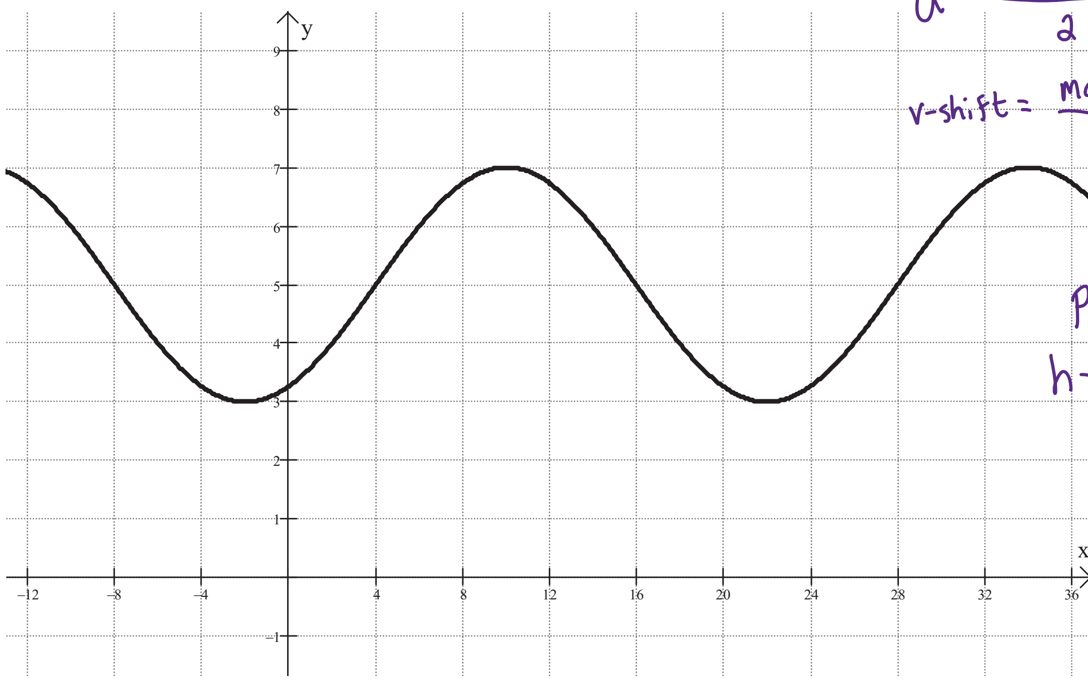
horizontal shift = π to the left



Max = v-shift + amp = 2+4 = 6

min = v-shift − amp = 2−4 = −2

19) Find one sine and one cosine equation for the following graph.



$a = \frac{\text{max} - \text{min}}{2} = \frac{7-3}{2} = 2$

v-shift $= \frac{\text{max} + \text{min}}{2} = \frac{7+3}{2} = 5$

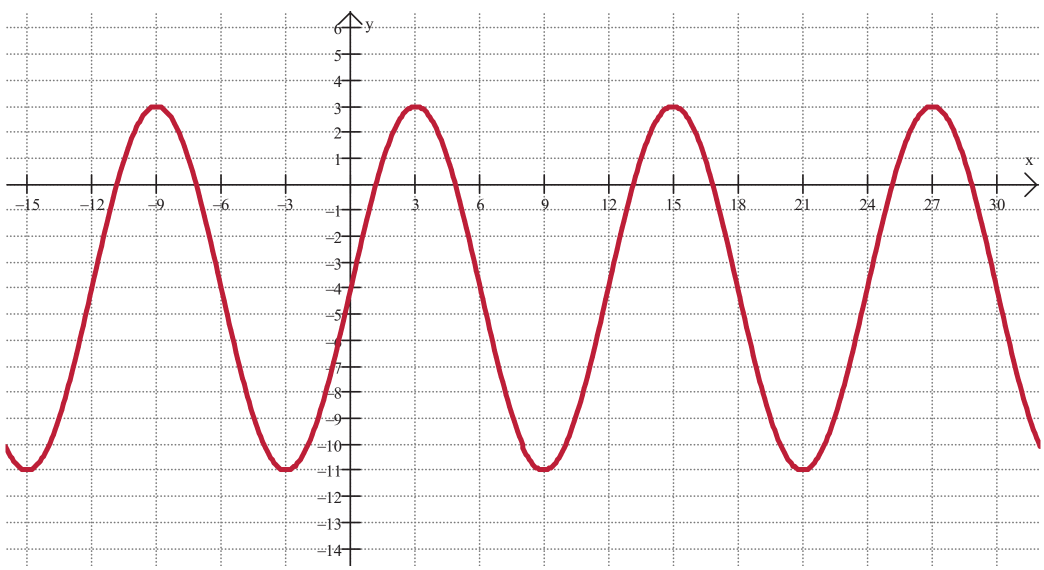
$p = 24 \Rightarrow b = \frac{2\pi}{24} = \frac{\pi}{12}$

h-shift for cosine 10 to the right

$y = 5 + 2\cos\left[\frac{\pi}{12}(x-10)\right]$ $\qquad$ $y = 5 + 2\sin\left[\frac{\pi}{12}(x-4)\right]$

h-shift is 1/4 period to the left of the h for cosine (in this case it's $\frac{1}{4} \cdot 24 = 6$)

20) Find one sine and one cosine equation for the graph below:



$a = \frac{\text{max} - \text{min}}{2} = \frac{3-(-11)}{2} = 7$

v-shift $= \frac{\text{max} + \text{min}}{2} = \frac{3-11}{2} = -4$

$p = 12 \Rightarrow b = \frac{2\pi}{12} = \frac{\pi}{6}$ $\quad$ h-shift for cosine 3 to the right

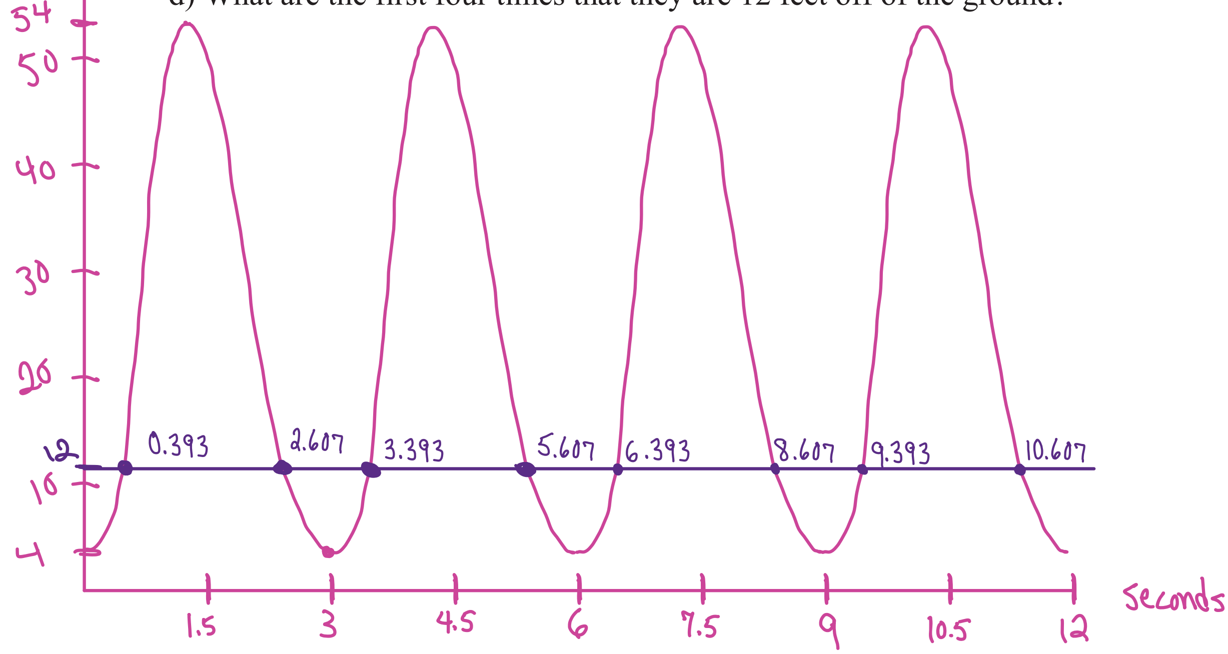
$y = -4 + 7\cos\left[\frac{\pi}{6}(x-3)\right]$

$y = -4 + 7\sin\left[\frac{\pi}{6}(x-0)\right]$

h-shift is 1/4 period to the left of the h for cosine (in this case it's $\frac{1}{4} \cdot 12 = 3$)

21) Seeking a peaceful place to study for finals, Uma and Tessa decide to go for a ride on the SkyStar wheel. Mikey, Will, Gavin, Bailey, and Monty, having all landed jobs as wheel operators, are fighting over whose turn it is to work the controls. In all the flailing and arguing, they inadvertently turn up the speed so that the wheel now makes one revolution every 3 seconds. Out of morbid curiosity, Miles and Jae recognize that the height of Uma and Tessa off the ground varies sinusoidally with time. They start timing when their car is at its lowest point 4 feet off of the ground. Since the diameter of the wheel is 50 feet,

a) Sketch at least one cycle of the graph that represents this situation.
b) Find an equation that represents this situation
c) How high off the ground are they at 7 seconds?
d) What are the first four times that they are 12 feet off of the ground?



b) $a = \dfrac{\text{max} - \text{min}}{2} = \dfrac{54-4}{2} = 25$

v-shift $= \dfrac{\text{max} + \text{min}}{2} = \dfrac{54+4}{2} = 29$

$p = 3 \Rightarrow b = \dfrac{2\pi}{3}$

h-shift for cosine
1.5 to the right

$y = 29 + 25\cos\left[\dfrac{2\pi}{3}(t-1.5)\right]$

c) $29 + 25\cos\left[\dfrac{2\pi}{3}(7-1.5)\right] = 41.5$ feet

d) $Y_1 = 29 + 25\cos\left[\dfrac{2\pi}{3}(t-1.5)\right]$

$Y_2 = 12$

$\boxed{\text{2nd}}$ $\boxed{\text{Trace}}$ → Intersect

$t = 0.393$ seconds
$= 2.607$ seconds
$= 3.393$ seconds
$= 5.607$ seconds

notice that they differ by the period which is 3