PRECALCULUS ACCELERATED NAME Solutions

Spring Final Exam Practice

On the exam, there will not be any MC questions. These are just for practice

A.M.D.G.

1. The equation of the line tangent to the graph of $f(x) = -x^2 + 4\sqrt{x}$ at the point where $x = 4$ is

(a) $y = -9x - 12$
(b) $y = -10x - 12$
(c) $y = -7x - 8$
(d) $y = -8x - 12$
(e) $y = -7x + 20$

Slope

$f(x) = -x^2 + 4x^{\frac{1}{2}}$

$f'(x) = -2x + 2x^{-\frac{1}{2}} = -2x + \frac{2}{\sqrt{x}}$

$f'(4) = -2(4) + \frac{2}{\sqrt{4}} = -8 + 1 = -7$

Point

$f(4) = -4^2 + 4\sqrt{4}$

$= -16 + 8$

$= -8$

Equation

$y + 8 = -7(x - 4)$

$y = -7x + 28 - 8$

$= -7x + 20$

2. If $y = \dfrac{-1}{3x^2 - 2}$, then $\dfrac{dy}{dx} = \dfrac{f'g - g'f}{g^2}$

(a) $\dfrac{-3x^2 + 6x + 2}{(3x^2 - 2)^2}$

(b) $\dfrac{-3x^2 + 6x + 2}{3x^2 - 2}$

(c) $\dfrac{6x}{(3x^2 - 2)^2}$

(d) $\dfrac{6x}{3x^2 - 2}$

(e) $-\dfrac{1}{6x}$

$= \dfrac{0(3x^2 - 2) - 6x(-1)}{(3x^2 - 2)^2} = \dfrac{6x}{(3x^2 - 2)^2}$

$f = -1 \qquad g = 3x^2 - 2$

$f' = 0 \qquad g' = 6x$

3. Which of the following is true about the function f if $f(x) = \dfrac{x^2 + x - 2}{2x^2 + x - 3}$?

I. f has a zero at $x = 1$. No
II. The graph of f has a POE at $x = 1$. yes
III. The graph of f has a vertical asymptote at $y = -\frac{3}{2}$. yes

$\dfrac{(x+2)\cancel{(x-1)}}{(2x+3)\cancel{(x-1)}}$

(a) II only (b) I and II only (c) I and III only (d) II and III only (e) I, II and III

4. Given these sign patterns, which of the following statements is/are true?

y	+	0	–	DNE	–	0	+
x		-2		0		$\frac{2}{3}$	

y'	+	DNE	+	0	–	DNE	–
x		-3		$\frac{1}{2}$		5	

Note: If the sign patterns were switched all three statements would be true

I. There is a zero at $x = \frac{1}{2}$ No

II. The function is increasing on $x \in (-\infty, -2) \cup \left(\frac{2}{3}, \infty\right)$ No

III. There is a maximum at $x = -2$ No

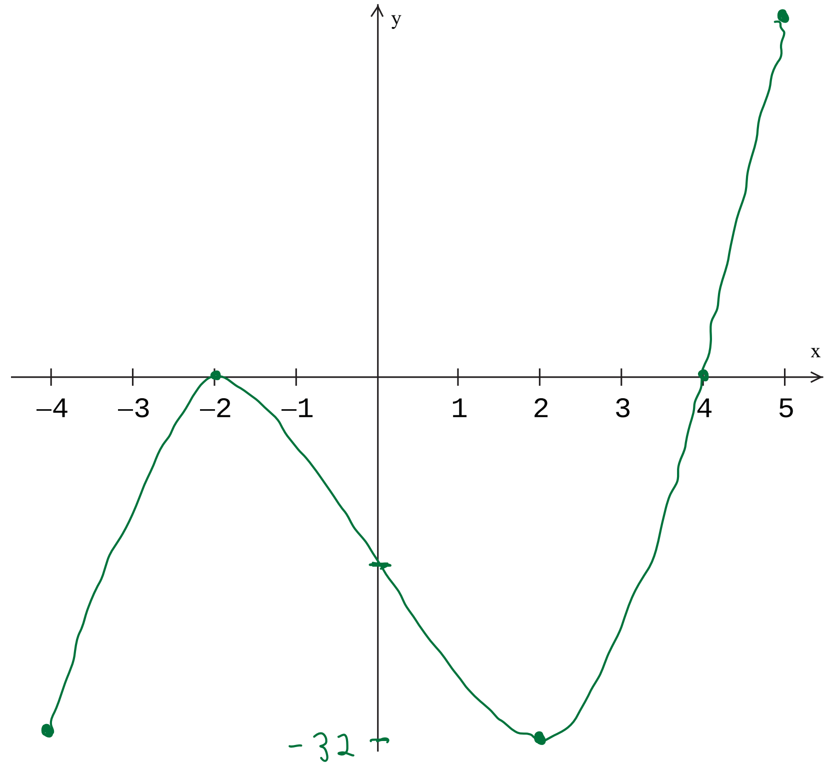
(a) I and III (b) II only (c) II and III (d) I, II, and III (e) None of these

5. $\lim\limits_{x \to 2} \dfrac{x^5 - 16x}{x^2 + 7x - 18} = \dfrac{0}{0} = \dfrac{x(x^4-16)}{(x+9)(x-2)} = \dfrac{x(x^2-4)(x^2+4)}{(x+9)(x-2)} = \dfrac{x\cancel{(x-2)}(x+2)(x^2+4)}{(x+9)\cancel{(x-2)}}$

(a) $\frac{64}{11}$ (b) 0 (c) undefined (d) $-\frac{64}{11}$ (e) $\frac{11}{64}$

6. Find the listed traits and graph $y = x^3 - 12x - 16$ over the domain $-4 \le x \le 5$.

x and y intercepts: y-int: $(0, -16)$



x-int: $0 = x^3 - 12x - 16$

-2	1	0	-12	-16
		-2	4	16
	1	-2	-8	0

$(x+2)(x^2 - 2x - 8)$

$(x+2)(x-4)(x+2) = 0 = (x+2)^2(x-4)$

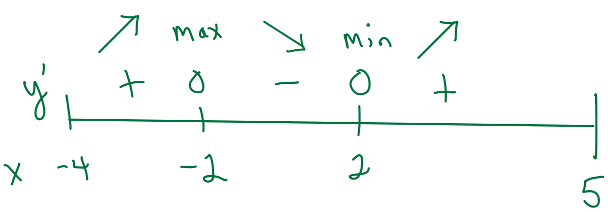
x-ints at $(-2, 0)$ $(4, 0)$

Critical Values: $y' = 3x^2 - 12 = 0$

$= 3(x^2 - 4)$

$= 3(x-2)(x+2) = 0$

$x = \pm 2$



Intervals of increasing/decreasing

(See sign pattern) increasing $x < -2$ or $x > 2$

decreasing $-2 < x < 2$

Extreme Points:

$x = 2$ $\quad y = 2^3 - 12(2) - 16 = 8 - 24 - 16 = -32$ $\quad (2, -32)$ absolute min

$x = -2$ $\quad y = (-2)^3 - 12(-2) - 16 = -8 + 24 - 16 = 0$ $\quad (-2, 0)$ relative max

$x = -4$ $\quad y = (-4)^3 - 12(-4) - 16 = -64 + 48 - 16 = -32$ $\quad (-4, -32)$ absolute min

$x = 5$ $\quad y = (5)^3 - 12(5) - 16 = 125 - 60 - 16 = 49$ $\quad (5, 49)$ absolute max

7. Find the given traits of of $f(x) = \frac{x^3+8}{x^2-4}$. $= \frac{\cancel{(x+2)}(x^2-2x+4)}{\cancel{(x+2)}(x-2)}$

y-intercepts: $f(0) = \frac{0+8}{0-4} = -2$

x-intercepts: $x^2 - 2x + 4 = 0 \Rightarrow$ does not factor   Hint: use quadratic formula

$$x = \frac{+2 \pm \sqrt{(-2)^2 - 4(4)}}{2} = \frac{2 \pm \sqrt{-16}}{2} \leftarrow \text{no solution}$$

VA: $x = 2$

POE: $x = -2$

Critical Values: $f = x^2 - 2x + 4$ $g = x - 2$

$f' = 2x - 2$ $g' = 1$

$$f'(x) = \frac{(2x-2)(x-2) - 1(x^2-2x+4)}{(x-2)^2}$$

$$= \frac{2x^2 - 4x - 2x + 4 - x^2 + 2x - 4}{(x-2)^2}$$

$$= \frac{x^2 - 4x}{(x-2)^2} = 0 \quad x = 0, 4$$

f'	+	0	−	DNE	−	0	+
x		0		2		4	

8. $\lim\limits_{x \to 5} \frac{\sqrt{x-2} - \sqrt{3}}{5 - x} = \frac{0}{0}$

Multiply top and bottom by conjugate $\rightarrow$ Example $(2 + \sqrt{x})(2 - \sqrt{x}) = 4 - x$

Difference of squares

$(a+b)(a-b) = a^2 - b^2$

$$\lim_{x \to 5} \frac{\sqrt{x-2} - \sqrt{3}}{5 - x} \cdot \frac{\sqrt{x-2} + \sqrt{3}}{\sqrt{x-2} + \sqrt{3}} = \lim_{x \to 5} \frac{(x-2) - (3)}{(5-x)(\sqrt{x-2} + \sqrt{3})} = \lim_{x \to 5} \frac{x-5}{(5-x)(\sqrt{x-2} + \sqrt{3})}$$

recall that $5 - 3 = -(3 - 5)$ so $(x - 5) = -(5 - x)$

$$\lim_{x \to 5} \frac{-\cancel{(5-x)}}{\cancel{(5-x)}(\sqrt{x-2} + \sqrt{3})} = \frac{-1}{\sqrt{5-2} + \sqrt{3}} = \frac{-1}{\sqrt{3} + \sqrt{3}} = \frac{-1}{2\sqrt{3}}$$

9. Jill and Elise are able to sell the idea of Mr Murphy being launched as a human cannonball for the Bruce Mahoney Rally provided that Mr. Murphy doesn't hit the ceiling of the gym which is 40 feet. Dylan and Annabel volunteer to figure out if the cannon they are using will not cause this to happen. They find that the equation for an object of equal weight being launched from the cannon at time t to be $h = 64t - 16t^2$ with $t = 0$ being the instant that Mr. Murphy is launched from the cannon.

a) What will Mr. Murphy's initial velocity be?

$h(t) = 64t - 16t^2$

64 ft/sec

or if you haven't taken physics just take the derivative to find velocity and plug in $t=0$

$v(t) = 64 - 32t \Rightarrow v(0) = 64$ ft/sec

b) Will Mr. Murphy hit the ceiling of the gym if launched at this velocity? How do you know?

Find his projected max height and see if it is higher than the ceiling

Max height $v(t) = 0$ (change of direction from rising to falling)

$v(t) = 64 - 32t = 0$

remember to look for a change in the velocity sign pattern to find a change of direction

$32t = 64$

$t = 2$ seconds

$h(2) = 64(2) - 16(2)^2$
$= 128 - 64 = 64$ ft

$\Rightarrow$ higher than 40 ft so he will hit the ceiling

c) If this were done on JB Murphy field without any concern of a ceiling, how long would Mr. Murphy be in the air?

When does he hit the ground?

$h(t) = 0 = 64t - 16t^2 = -16t(t-4) = 0$

He is at height zero at 0 and 4 seconds

so he is in the air for 4 seconds

10. Solve for x

a) $\log x + \log(x-48) = 2$

remember that $\log = \log_{10}$

$\log x(x-48) = 2 \Rightarrow \log_{10} n = p$ means that $10^p = n$

$x(x-48) = 10^2$

$x^2 - 48x = 100$

$x^2 - 48x - 100 = 0 = (x-50)(x+2) = 0 \quad x = 50, \cancel{-2}$ only $x=50$ works as a solution

b) $4^{x+2} = 8^{x-3}$

$(2^2)^{x+2} = (2^3)^{x-3}$

$2^{2x+4} = 2^{3x-9}$

$2x+4 = 3x-9$

$-x = -13 \Rightarrow x = 13$

Alternate Method

$\ln 4^{x+2} = \ln 8^{x-3}$

$(x-2)\ln 4 = (x-3)\ln 8$

$x-2 = (x-3)\frac{\ln 8}{\ln 4}$

$x-2 = (x-3)\log_4 8$

$4^p = 8?$ $\quad p = \frac{3}{2}$ because $(\sqrt{4})^3 = 8$

$x-2 = (x-3)\frac{3}{2}$

$x-2 = \frac{3}{2}x - \frac{9}{2}$

$\frac{13}{2} = \frac{1}{2}x$

$13 = x$

11. Find the derivative of the given functions

a) $f(x) = e^{\sqrt{x^2-1}} = e^{(x^2-1)^{\frac{1}{2}}}$

$f'(x) = e^{(x^2-1)^{\frac{1}{2}}}\ \frac{1}{\cancel{2}}(x^2-1)^{-\frac{1}{2}}\ \cancel{2}x$

$= e^{(x^2-1)^{\frac{1}{2}}}\ \frac{x}{(x^2-1)^{\frac{1}{2}}} = \frac{x e^{\sqrt{x^2-1}}}{\sqrt{x^2-1}}$

b) $g(x) = \log\sqrt[3]{x^4+6x} = \log(x^4+6x)^{\frac{1}{3}} = \frac{1}{3}\log(x^4+6x)$

$g'(x) = \frac{1}{3}\ \frac{1}{(x^4+6x)\ln 10}\cdot 4x^3+6 = \frac{4x^3+6}{3(x^4+6x)\ln 10}$

c) $h(x) = \ln\sqrt[4]{e^x} = \ln(e^x)^{\frac{1}{4}} = \frac{1}{4}\cancel{\ln e}^x = \frac{1}{4}x$

$h'(x) = \frac{1}{4}$

or

$h(x) = \ln e^{\frac{1}{4}x}$

$h'(x) = \frac{1}{\cancel{e^{\frac{1}{4}x}}}\cdot \cancel{e^{\frac{1}{4}x}}\cdot\frac{1}{4} = \frac{1}{4}$

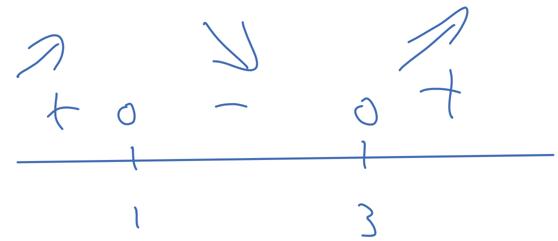
12. Find all traits and sketch $f(x) = x^3 - 6x^2 + 9x - 4$.

Zeros: $x^3 - 6x^2 + 9x - 4 = 0$ $\quad x = 1, 4$

y-intercept: $(0, -4)$

intervals of increasing/decreasing

$$3x^2 - 12x + 9 = 0$$
$$3(x^2 - 4x + 3) = 0$$
$$(x-3)(x-1) = 0$$



increasing $x < 1$ or $x > 3$

decreasing $1 < x < 3$

Extreme Points:

$f(1) = 0$ (we already determined that 1 is an x-intercept)

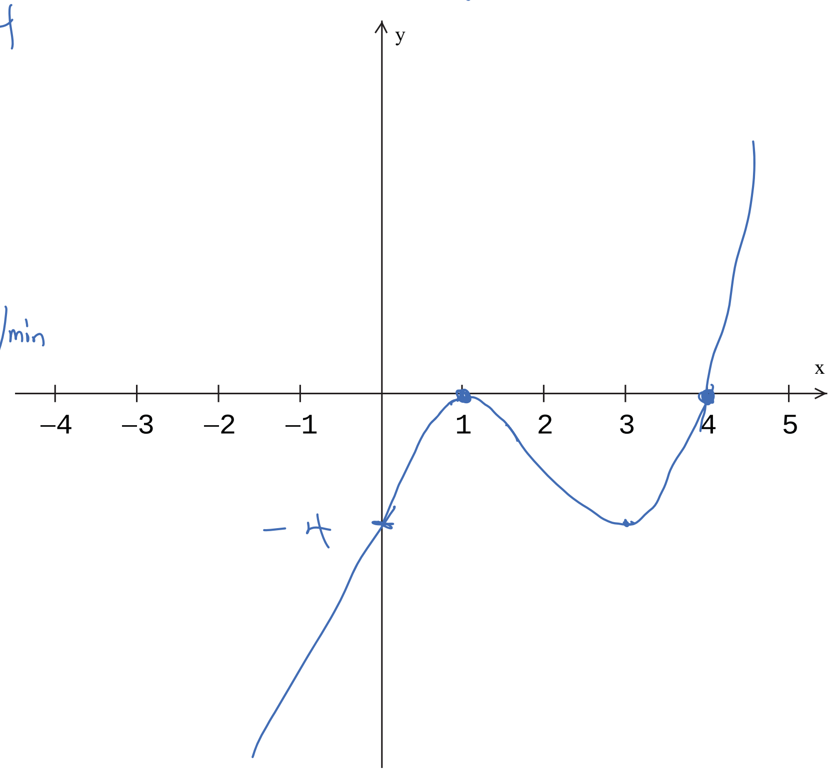
all real numbers

$$f(3) = 3^3 - 6(3)^2 + 9(3) - 4$$
$$= 27 - 54 + 27 - 4$$
$$= -4$$

relative min at $(3, -4)$

relative max at $(1, 0)$

no absolute max/min



Standard 3b	Factor polynomials using grouping and sum/difference rules
Standard 4f	Factor polynomials to find zeros algebraically using synthetic substitution.
Standard 5a	Evaluate limits involving the indeterminate form 0/0
Standard 5d	Use the limit definition to find the derivative of a polynomial function
Standard 5e	Find the equation of the tangent and normal lines to a polynomial function at a given point
Standard 5g	Given the position function of an object as a polynomial, use the derivative to find the velocity and acceleration functions
Standard 5h	Use sign patterns to describe the motion of an object
Standard 6a	Use the derivative to find the critical values of a polynomial
Standard 6b	Use sign patterns to determine the intervals where a function is increasing or decreasing
Standard 6c	Identify the type of extreme point represented by a particular critical value
Standard 6e	Sketch a polynomial graph using the traits of Domain, x and y intercepts, End Behavior, Extreme Points, and Range
Standard 7a	Find the zeros, y-intercept, Vertical Asymptotes, and POE's of a rational function
Standard 7f	Use the Quotient Rule to find critical values of a rational function
Standard 8b	Find the derivative of a composite function using the Chain Rule
Standard 10a	Solve equations involving exponential and/or logarithmic functions
Standard 10e	Find derivatives and extremes of log and exponential functions