AP Statistics | **Name:** Solutions
Mr Murphy | **Date:**
Sampling Distributions Sample Test | **Period:**

Multiple Choice

1. According to the Central Limit Theorem which of the following are true?

I. The larger the sample, the larger the spread in the sampling distribution.
II. The larger the sample, the larger the center in the sampling distribution.
III. The larger the sample, the closer to normal is the shape of the sampling distribution.

(a) I and II (b) I and III (c) III only (d) I, II, and III (e) None of the above

2. The Burger Joint believes that 80 percent of all students on a particular campus prefer their hamburgers served at the campus grill. The owner of the Burger Joint decides to give a taste to a sample of 225 students. What is the probability that at least 183 of the students in the sample prefer the hamburger from the campus grill?

Assumptions

(1) $np = 225(.8) = 180 \geq 10$
(2) $n(1-p) = 225(0.2) = 45 \geq 10$
(3) SSSRTP or 225 is less than 10% of population

$P(x>183) = P\left(p > \frac{183}{225}\right)$
$= \text{normalcdf}\left(\frac{183}{225}, 1E99, 0.8, \sqrt{\frac{(0.8)(0.2)}{225}}\right)$
$= 0.3085$

(a) 0.1615
(b) 0.3085
(c) 0.8385
(d) 0.6915
(e) 1.0217

3. John is an expert horseshoe thrower who misses only 15% of the time. Choose the expression that correctly represents the probability John will miss fewer than 50 times if he throws 400 horseshoes.

$\rightarrow \mu = \overset{p}{0.15}(\overset{n}{400}) = 60$

$P\left(p < \frac{50}{400}\right) = \text{normalcdf}(-1E99, 50, 60, 7.14)$

Assumptions
(1) $np = 60 \geq 10$
(2) $n(1-p) = 340 \geq 10$
(3) $400 < 10\%$ of all his throws

$\sigma_{\hat{p}} = \sqrt{\frac{(.15)(.85)}{400}} \Rightarrow \sigma_{\bar{x}} = 400(\sigma_{\hat{p}})$
≈ 7.14

See last page for further explanation

(a) normalcdf(-E99, 50, 60, 7.14)
(b) normalcdf(50, E99, 60, 7.14)
(c) normalpdf(-E99, 50, 400, 7.14)
(d) normalpdf(-E99, 50, 60, 7.14)
(e) normalcdf(-E99, 50, 400, 7.14)

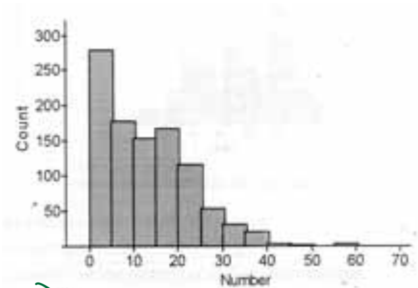
4. A tire manufacturer claims that its tires will last an average of 40,000 miles with a standard deviation of 3,000 miles. Forty-nine tires were placed on test, and the average failure was recorded. The probability that the average value of failure miles is between 39,500 and 40,000 is

$\mu = 40{,}000$
$\sigma = 3000$
$n = 49 \ (\geq 30 \checkmark)$

(a) 0.1790
(b) 0.8790
(c) 0.1210
(d) 0.6210
(e) 0.3790

$P(39{,}500 < x < 40000) = \text{normalcdf}\left(39{,}500, 40000, 40000, 3000/\sqrt{49}\right)$

5. Shown below is a distribution with mean 12.262 and standard deviation 9.610

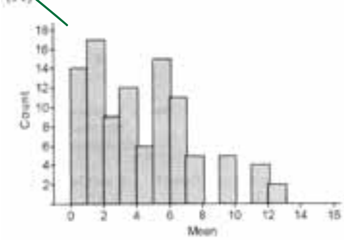


One hundred samples of size 9 are drawn from this population, and the sample means are recorded. Which of the following is most likely to represent this distribution of sample means?

$\bar{X} = 12.262 \qquad \sigma_{\bar{x}} = \frac{9.610}{\sqrt{9}} = \frac{9.610}{3} = 3.203$

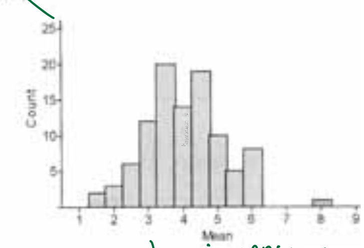
Expect a shape with a slightly rightward skew because the sample size is only $n = 9$

~~(A)~~



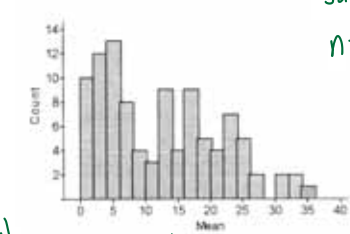
center is not 12

~~(B)~~



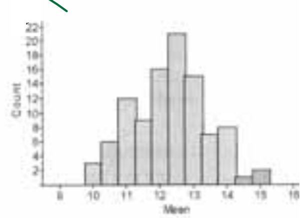
shape is approx normal

~~(C)~~



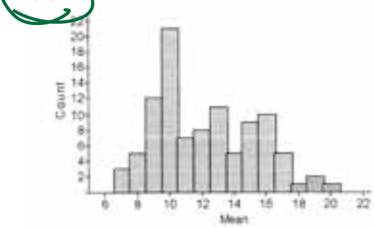
s.d. is too large

~~(D)~~



Shape is approx normal

(E)



6. A population that is bimodal and is skewed to the left has a mean of 120 and a standard deviation of 32. What is the probability that a sample of size 60 will have a mean less than 110?

(a) 0.0059
(b) 0.0078
(c) 0.0102
(d) 0.3783
(e) Cannot be determined since the population is not normal

$P(\bar{x} < 110) = \text{normalcdf}(-1E99, 110, 120, 32/\sqrt{60})$

7. Heights of fourth graders are normally distributed with a mean of 52 inches and a standard deviation of 3.5 inches. For a research project, you plan to measure a simple random sample of 30 fourth graders. For samples such as yours, 10% of the samples should have an average height below what number?

(a) 47.52 inches
(b) 51.18 inches
(c) 51.85 inches
(d) 52.82 inches
(e) 56.48 inches

$x = \text{invNorm}(0.1, 52, 3.5/\sqrt{30})$

Free Response (4 pts. each)

1. (2007 Q3) Big Town Fisheries recently stocked a new lake in a city park with 2,000 fish of various sizes. The distribution of the lengths of these fish is approximately normal.

(a) Big Town Fisheries claims that the mean length of the fish is 8 inches. If the claim is true, which of the following would be more likely?

• A random sample of 15 fish having a mean length that is greater than 10 inches

or

• A random sample of 50 fish having a mean length that is greater than 10 inches

Justify your answer.

The standard deviation of the sampling distribution will be smaller for $n=50$ because $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ which means that the spread of the $n=15$ distribution will be wider

example

We can assume that σ is anything so we'll just assign it a value $\sigma = 1$ inch

$\text{normalcdf}\left(10, 1E99, 8, \frac{1}{\sqrt{15}}\right) = 0.0049$

$\text{normalcdf}\left(10, 1E99, 8, \frac{1}{\sqrt{50}}\right) = 0.00000122$

(b) Suppose the standard deviation of the sampling distribution of the sample mean for random samples of size 50 is 0.3 inch. If the mean length of the fish is 8 inches, use the normal distribution to compute the probability that a random sample of 50 fish will have a mean length less than 7.5 inches.

$\mu_{\bar{x}} = 8 \quad \sigma_{\bar{x}} = 0.3 \ (n=1)$

$P(\bar{x} < 7.5) = \text{normalcdf}(-1E99, 7.5, 8, 0.3) \approx 0.048$

(c) Suppose the distribution of fish lengths in this lake was non-normal but had the same mean and standard deviation. Would it still be appropriate to use the normal distribution to compute the probability in part (b)? Justify your answer.

Yes, CLT says that 50 is large enough $(50 \geq 30)$

2. Some biologists believe the evolution of handedness is linked to complex behaviors such as tool-use. Under this theory, handedness would be genetically passed on from parents to children. That is, left-handed parents would be more likely to have left-handed children than right-handed parents. An alternate theory asserts that handedness should be random, with left- and right-handedness equally likely. In a recent study using a simple random sample of $n = 76$ right-handed parents, 50 of the children born were right-handed.

Suppose handedness is a random occurrence with either hand equally likely to be dominant, implying that the probability of a right-handed offspring is $p = 0.5$.

(a) Show that it is reasonable to approximate the sampling distribution of *p* using a normal distribution.

$p = 0.5 \quad n = 76$

Check assumptions

① $np \geq 10$

② $n(1-p) \geq 10$

③ SSSRTP — 10% rule

$\hat{p} = 0.5 \quad \sigma_{\hat{p}} = \sqrt{\frac{(0.5)(1-0.5)}{76}}$

(b) Assuming left- and right-handed children are equally likely from right-handed parents, what is the probability of observing at least 50 right-handed children in a sample of 76?

$P(X > 50) = P\left(\hat{p} > \frac{50}{76}\right)$

$\text{normalcdf}\left(\frac{50}{76}, 1E99, 0.5, \sigma_{\hat{p}}\right) \approx 0.003$

This indicates that left/right handedness is

not random but genetic

$$\sigma_{\hat{p}} = \sqrt{\frac{(.15)(.85)}{400}} \Rightarrow \sigma_{\bar{x}} = 400(\sigma_{\hat{p}})$$

$$\approx 7.14$$

To treat this problem as a sample proportion, we would do it like this:

$$P\left(p < \frac{50}{400}\right) = \text{normalcdf}\left(-1E99, \frac{50}{400}, 0.15, 0.01785\right)$$

but since your answer choices are all presented as sample means, you can just multiply through by the sample size which is 400

$$P\left(p < \frac{50}{400}\right) = \text{normalcdf}\left(-1E99, \frac{50}{400}, 0.15, 0.01785\right)$$

multiply each by 400

$$P\left(p < \frac{50}{400}\right) = \text{normalcdf}\left(-1E99, 50, 60, 7.14\right)$$