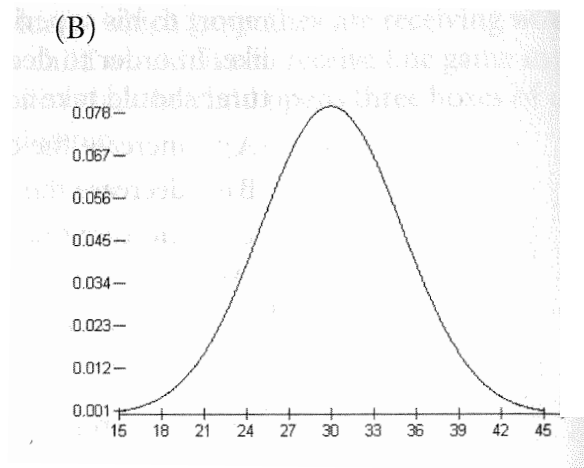
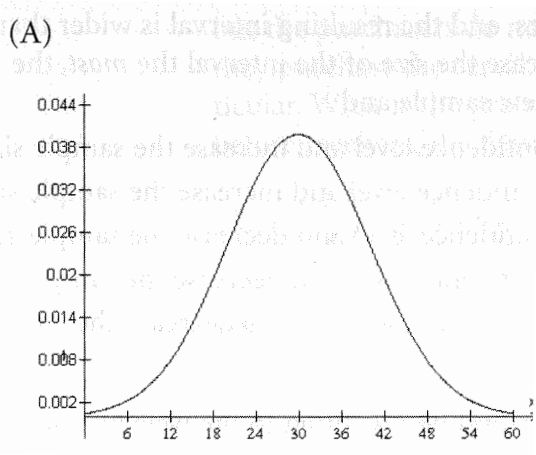


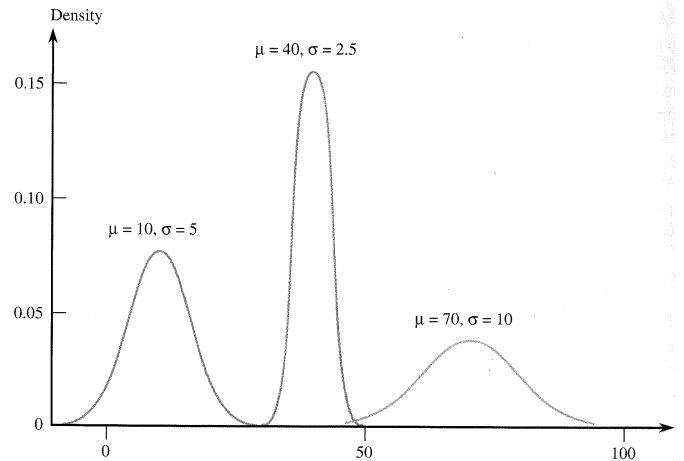
- Goals:
1. Calculate probabilities from a standard normal curve.
  2. Calculate probabilities from any normal curve.
  3. Given a probability, find its corresponding  $x$  value/ $z$  score.
  4. Live in the moment.

**Normal distributions** are continuous probability distributions that are bell shaped and symmetric.

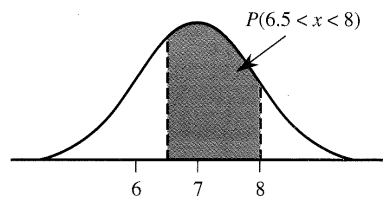
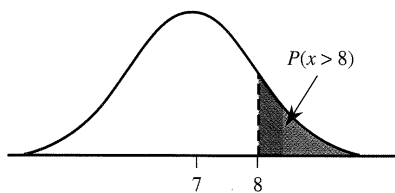
Ex1 Which of the following distributions has a mean of 30 and a standard deviation of 5?



There are many different normal distributions. They are distinguished from one another by their mean,  $\mu$ , and their standard deviation,  $\sigma$ .

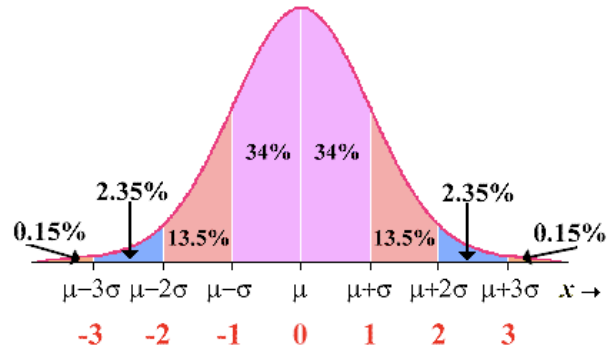


Use these distributions to calculate probabilities.



- The **standard normal distribution** is the normal distribution with  $\mu = 0$  and  $\sigma = 1$ . It is customary to use the letter  $z$  to represent a variable whose distribution is described by the standard normal curve.
- Recall from Section 2.4:  $z$  scores and the Empirical Rule

$$z \text{ score} = \frac{\text{value} - \text{mean}}{\text{standard deviation}}$$



Ex2 In this year's county mathematics competition, a student scored 40; in last year's competition, the student scored 35. The average score this year was 38 with a standard deviation of 2. Last year's average score was 34 with a standard deviation of 1. In which year did the student score better?

- The student scored better on this year's exam.
- The student scored better on last year's exam.
- The student scored equally well on both exams.
- Without knowing the number of test items, it is impossible to determine the better score.
- Without knowing the number of students taking the exam in the county, it is impossible to determine the better score.

Ex3 The weights of women are approximately normally distributed. This week, the  $z$  scores of weight for a member of a weight-watching group is 1.25. Which of the following is a correct interpretation of this  $z$ -score?

- This week the member weighs 1.25 lbs. more than last week.
- This week the member weighs 1.25 lbs. less than last week.
- This week the member weighs 1.25 lbs. more than the average woman.
- This week the member weighs 1.25 standard deviations more than she did last week.
- This week the member weighs 1.25 standard deviations more than the average woman.

Ex4 Calculate the following probabilities with your calculator.

(a)  $P(z < -1.76) =$

(b)  $P(z \leq 0.58) =$

(c)  $P(-2 < z < 2) =$

(d)  $P(z > 1.18) =$

(e)  $P(z > 1.96) =$

(f)  $P(-1.76 < z < 0.58) =$

Calculator: 2nd -> Vars -> 2 -> lower bound, upper bound, 0, 1  
Display: **normalcdf(lower bound, upper bound, 0, 1)**  
Note:  $\infty = 1E99$  ,  $-\infty = -1E99$

**Finding Probabilities for any Normal Distribution**

Ex5 The growth of children can be an important indicator of general levels of nutrition and health. Data suggest that a reasonable model for the probability distribution of the continuous numerical variable  $x =$  height of a randomly selected 5-year old child is a normal distribution with mean  $\mu = 100$  cm and standard deviation  $\sigma = 6$  cm. What proportion of the heights is between 94 and 112 cm?

What is the probability that a randomly chosen child will be taller than 110 cm?

Calculator: 2nd -> Vars -> 2 -> lower bound, upper bound, mean, s.d.  
Display: **normalcdf(lower bound, upper bound, mean, s.d)**  
Note:  $\infty = 1E99$  ,  $-\infty = -1E99$

Ex6 Although there is some controversy regarding the appropriateness of IQ scores as a measure of intelligence, IQ scores are commonly used for a variety of purposes. One commonly used IQ scale has a mean of 100 and a standard deviation of 15, and IQ scores are approximately normally distributed. If we define the random variable  $x = \text{IQ score of a randomly selected individual}$ , then  $x$  has approximately a normal distribution with  $\mu = 100$  and  $\sigma = 15$ .

One way to become eligible for Mensa, an organization purportedly for those of high intelligence, is to have an IQ score above 130.

(a) What proportion of the population would qualify for Mensa membership?

(b) What proportion of the population has IQ scores below 80?

(c) What proportion of the population has IQ scores between 75 and 125?

Ex7 An electronic product takes an average of 3.4 hours to move through an assembly line. If the standard deviation is 0.5 hours, what is the probability that an item will take between 3 and 4 hours? Assume a normal distribution.

- (a) 0.2119
- (b) 0.2295
- (c) 0.3270
- (d) 0.3811
- (e) 0.6730

### The Backwards “Situation”

- Sometimes you will be given a probability and asked for an  $x$  value or a  $z$  score. Use **InvNorm** on your calculator or the formula to solve the problem.

#### Calculator

- **InvNorm** : 2nd -> Vars -> 3 -> percentile -> mean -> s.d.
- Display: **invNorm(percentile, mean, s.d.)**

#### Formula

- To convert a  $z$  score back to an  $x$  value, use
$$x = \mu + z\sigma$$

Ex8 Data on the length of time required to complete registration for classes using a telephone registration system suggest that the distribution of the variable  $x =$  time to register for students at a particular university can be well approximated by a normal distribution with mean  $\mu = 12$  min and standard deviation  $\sigma = 2$  min.

Because some students do not sign off properly, the university would like to disconnect students automatically after some amount of time has elapsed. It is decided to choose this time such that only 1% of the students are disconnected while they are still attempting to register.

What is the cutoff time for disconnection?

Ex9 A sales person ranks in the top 5% of all sales people in a large company. If the annual mean sales amount is \$750,000 and the standard deviation is \$150,000, how much does the person sell each year? Assume a normal distribution.

- (a) \$757,500
- (b) \$996,750
- (c) \$1,044,000
- (d) \$1,697,100
- (e) \$2,650,500

**Checkpoint:**

**Multiple Choice**

1. A factory dumps an average of 2.43 tons of pollutants into a river every week. If the standard deviation is 0.88 tons, what is the probability that in a week more than three tons are dumped? Assume a normal distribution.

- (a) 0.2585
- (b) 0.2843
- (c) 0.6500
- (d) 0.7157
- (e) 0.7422

2. Runners competed in a local road race. The mean finishing time for the race was 43.5 minutes with a standard deviation of 16.2 minutes. The sponsors wanted to have a special race for those who were in the fastest 10%. Assuming the times were normally distributed, which of the following is the cutoff time?

- (a) 22.7 minutes
- (b) 25.7 minutes
- (c) 39.2 minutes
- (d) 42.2 minutes
- (e) 64.3 minutes

3. Which of the following are true?

- I. The area under a normal curve is always equal to 1, no matter what the mean and standard deviation are.
- II. The smaller the standard deviation of a normal curve, the higher and narrower the graph.
- III. Normal curves with different means are centered around different numbers.

- (a) I and II      (b) I and III      (c) II and III      (d) I, II, and III      (e) None of the above

4. A fire department in a rural county reports the mean response is 22 minutes. A home owner was told the response time of 30 minutes to his neighborhood was at the 3rd quartile. What standard deviation did the report use if the times were normally distributed?

- (a) 2.7 minutes
- (b) 3.4 minutes
- (c) 7.1 minutes
- (d) 11.9 minutes
- (e) 16.0 minutes