Rules of Logarithmic and Exponential Functions

$5^3 = x$ $x^3 = 125$ $5^x = 125$

Which of these would be hardest to solve algebraically?

$5^3 = 125$ $\sqrt[3]{x^3} = \sqrt[3]{125}$ $5^x = 125$

$x = 5$ **?**

We do have a function that we can perform here that would actually help us solve for an <u>unknown exponent</u>.

$$5^x = 125 \longrightarrow \log_5 125 = x$$

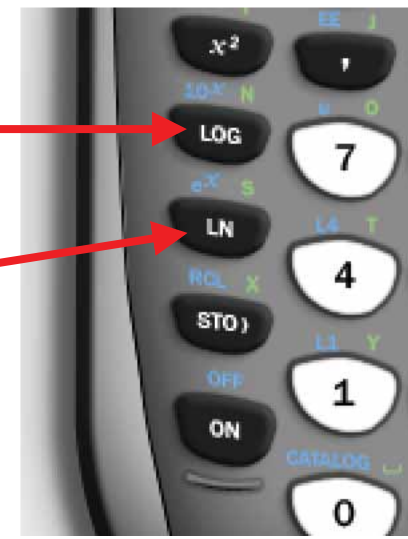
Logarithmic Functions

$$\log_5 125 = x$$

Notice the two log buttons on your calculator:

$\log_{10}$

$\log_e$



The number e has a significance, trust me

Examples:

Ex 1) $\log_3 81 = 4$

$$3^4 = 81$$

Ex 2) a) $9^{\frac{3}{2}} = 27$

$$\log_9 27 = \frac{3}{2}$$

b) $16^{-\frac{3}{4}} = \frac{1}{8}$

$$\log_{16} \frac{1}{8} = -\frac{3}{4}$$

Did You Know that this means this?

$$\log_{16}\frac{1}{8} = -\frac{3}{4}$$

$$\log_{16} 8 = \frac{3}{4}$$

We will find out why shortly

Laws of Logs

$$\log_a M + \log_a N = \log_a MN$$

Really?

$$\log_2 4 + \log_2 8 = \log_2 32$$

$$2 + 3 = 5 \quad \checkmark$$

$$\log_a M - \log_a N = \log_a \frac{M}{N}$$

$$\log_a M + \log_a M + \log_a M$$

$$\log_a M + \log_a M + \log_a M$$

and therefore...

$$\log_a M^n = n \log_a M$$

Did You Know This?

$$\log_{16}\frac{1}{8}=-\frac{3}{4}$$

$$\log_{16}8=\frac{3}{4}$$

$$\log_a M^n = n\log_a M$$

$$\log_{16}8^{-1}=-\frac{3}{4} \longrightarrow (-1)\log_{16}8=-\frac{3}{4}$$

$$\log_{16}8=\frac{3}{4}$$

Ex 3

$$\frac{1}{3}\log_4 64 - 4\log_4 2$$

$$\log_4 64^{\frac{1}{3}} - \log_4 2^4$$

$$\log_4 4 - \log_4 16$$

$$\log_4 \frac{4}{16}$$

$$\log_4 \frac{1}{4} \quad = \log_4 4^{-1} \quad = (-1)\log_4 4$$

$$= -1$$

$$\log_{17} 1,419,857 = x$$

$$17^x = 1,419,857 \longrightarrow 17^5 = 1,419,857$$

Only the most recent versions of the TI-84 have a log function with a variable base.

But how would we do this on older calculators?

How about using a log function that is on there like base 10?

$$\log_{10} 17^x = \log_{10} 1,419,857$$

$$x \log_{10} 17 = \log_{10} 1,419,857$$

$$x = \frac{\log_{10} 1,419,857}{\log_{10} 17} = 5$$

Change of Base Formula

$$\log_a M = \frac{\log_b M}{\log_b a}$$

$$\log_{17} 1,419,857 = x$$

$$17^x = 1,419,857 \qquad\qquad 17^5 = 1,419,857$$

These two equations say the same thing which tells us something:

The log and exponential functions are inverses of each other

This also tells us that algebraically, each can be used to "undo" the other

What do I mean by this? Just watch…

$$17^5 = 1,419,857$$

$$\log_{17} 17^5 = \log_{17} 1,419,857$$

$$5 = \log_{17} 1,419,857$$

We can also do this

$$\log_{17} 1,419,857 = x$$

$$17^{\log_{17} 1,419,857} = 17^x$$

$$1,419,857 = 17^x$$

This gives us these two expressions to remember

$$\log_a a^x = x$$

and

$$a^{\log_a x} = x$$

So let's recap…

$$\log_5 125 = x$$

$$\log_a M + \log_a N = \log_a MN$$

$$\log_a M - \log_a N = \log_a \frac{M}{N}$$

$$\log_a M^n = n \log_a M$$

$$\log_a a^x = x$$

$$a^{\log_a x} = x$$

And for the calculator…

$$\log_a M = \frac{\log_b M}{\log_b a}$$

Which can be done using the two programmed bases

$$\log_{10} \ \ and \ \ \ln$$

More on this one soon…

Ex 4

$$\log_3(x+4)+\log_3(2x-7)=3$$

$$\log_3(x+4)(2x-7)=3$$

$$(x+4)(2x-7)=3^3$$

$$2x^2+x-28=27$$

$$2x^2+x-55=0$$

$$(2x+11)(x-5)=0$$

$$x=-\frac{11}{2},\ 5$$

$x = 5$ can be shown clearly to be a solution but…

try this other solution on your calculator

Ex 4 $\log_3(x+4)+\log_3(2x-7)=3$

$$x=-\frac{11}{2},\ 5$$

x = 5 can be shown clearly to be a solution but…

try this other solution on your calculator

$$\log_3\left(-\frac{11}{2}+4\right)+\log_3(-11-7)=3$$

$$\log_3\left(-\frac{3}{2}\right)+\log_3(-18)=3$$

You will see the reason for this shortly…

Try these on your calculator

$\log 10 =$

$\log 100 =$

$\log 1000 =$

$\log 5 =$

```
log(10)
                 1
log(100)
                 2
log(1000)
                 3
log(5)
      .6989700043
```

$\log(-5) =$

```
ERR:NONREAL ANS
1:Quit
2:Goto
```

But why?

$\log_a x$

only for all $x > 0$